BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI

Volumul 62 (66)

Numărul 1

MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

2016 **Editura POLITEHNIUM**

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
PUBLISHED BY
“GHEORGHE ASACHI” TECHNICAL UNIVERSITY OF IAŞI
Editorial Office: Bd. D. Mangeron 63, 700050, Iaşi, ROMÂNIA
Tel. 40-232-278683; Fax: 40-232-237666; e-mail: polytech@mail.tuiasi.ro

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
BULLETIN OF THE POLYTECHNIC INSTITUTE OF IAŞI
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APPROXIMATION PROPERTIES FROM A MATHEMATICAL-PHYSICAL PERSPECTIVE. POSSIBLE CORRELATIONS WITH THE NEURONAL NETWORK FRACTALITY

BY

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Received: January 11, 2016
Accepted for publication: April 7, 2016

Abstract. In this paper, we consider different problems concerning regularity viewed as an approximation property in several hit-and-miss hypertopologies as a foundation for self-similarity and fractality from a mathematical - physical perspective. Since in some examples of fractals, as the neuronal network or the circulatory system, the uniform property of the Hausdorff hypertopology is not appropriate, the Wijsman hypertopology may be preferred because it could describe better the pointwise properties of fractals.

Keywords: hit-and-miss hypertopologies; Hausdorff topology; Wijsman topology; regularity; approximations; fractal theories; non-differentiable physics; Scale Relativity Theory.

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1. Introduction

Hypertopologies represent an useful tool in optimization, convex analysis, economics, image processing, sound analysis and synthesis and many other fields. In this sense, important theoretical and practical results have been established (Beer, 1993; Apreutesei, 2003; Hu & Papageorgiou, 1997) - concerning Vietoris topology, (di Lorenzo & di Maio, 2006) - melodic similarity, Lu *et al*. (2001) - word image matching, involving Hausdorff-Pompeiu metric. Approaches of topology in psychology have been also obtained (Lewin *et al.*, 1936; Brown, 2012).

All hypertopologies known so far are of the hit-and-miss type, which led to their unification under a single one - the Bombay Hypertopology (di Maio & Naimpally, 2008).

The idea of modeling phenomena behavior at multiple scales has become a useful tool in pure and applied mathematics and physics. Fractal-based techniques lie at the heart of these areas, since fractals are multi-scale objects, which often describe such phenomena better than traditional mathematical models. Hyperspace theories concerning the Hausdorff metric and the Vietoris topology have been developed as a foundation for self-similarity and fractality (Kunze *et al*., 2012; Wicks, 1991).

Topological methods facilitate the study of the dynamical systems chaotic nature (Sharma & Nagar, 2010; Wang *et al*., 2009; Goméz-Rueda *et al*., 2012; Li, 2012; Liu *et al.*, 2009; Ma *et al*., 2009; Fu & Xing, 2012), since they are collective phenomenon emerging out of many segregated components. Most of these systems are collective (set-valued) dynamics of many units of individual systems. The underlying dynamics of the dynamical systems is set-valued, collective (Edalat, 1995; Gavriluț & Agop, 2013).

Since many phenomena with complex patterns and structures are widely observed in brain, this permits the mathematical modeling and analysis of neuronal networks from the dynamical systems set-valued viewpoint. These phenomena are some manifestations of a multidisciplinary paradigm called emergence or complexity. They share a common unifying principle of dynamic arrays. Precisely, interconnections of a sufficiently large number of simple dynamic units can exhibit complex and self-organizing behaviors. System's complexity can be measured by the topological entropy of the topological dynamical system. In this sense, it has been proposed a new definition of the topological entropy for continuous mappings on arbitrary topological spaces (Liu *et al.*, 2009).

In recent years, in computational graphics and automatic recognition of figures problems, it was necessary to measure accurately the matching, *i.e.* to calculate the distance between two sets of points. So, one should use an acceptable distance which has to satisfy the first condition in the definition of a distance - the distance is zero if and only if the overlap is perfect. An appropriate metric is the Hausdorff metric measuring the degree of overlap of two compact sets. In some examples of fractals (the neuronal networks, the circulatory system), Hausdorff topology is not appropriate due to its uniform property. As a more convenient topology on the set of values of the multifunctions which we study, Wijsman topology may be preferred instead of the Hausdorff one, because Wijsman topology could describe better the pointwise properties of fractals. Recently, generalized fractals in hyperspaces endowed with Hausdorff, or with Vietoris hipertopology have been studied (Andres & Fišer, 2004; Andres & Rypka, 2012; Banakh & Novosad, 2013; Kunze *et al*., 2012).

On the other hand, together with the increasing interest in hypertopologies, non-additive set-valued measure theory continued to develop. In this context, regularity is known as an important continuity property with respect to different topologies. At the same time, it can be viewed as an approximation property of different "unknown" sets by other sets for which we have more information (Gavriluț & Agop, 2015a; Gavriluț & Agop, 2015b). From a mathematical perspective, this approximation is usually done from the left by closed sets (or more restrictive, by compact sets) and/or from the right by open sets. As a theoretical application of regularity, classical Lusin theorem concerns with the existence of continuous restrictions of measurable functions and it is a very important and useful result for discussing different kinds of approximation of measurable functions defined on special topological spaces. It has applications in the study of convergence of sequences of Sugeno and Choquet integrable functions. Regular Borel measures are important in studying Kolmogorov fractal dimensions (Barnsley, 1988; Mandelbrot, 1983).

As it is well known, artificial neural networks are inspired by the biological nervous system, in particular, the human brain and one of the most interesting characteristics of the human brain is its ability to learn. An application of Lusin's theorem in the study of the approximation properties of neural networks was established (Li *et al*., 2007) since the learning ability of a neural network is closely related to its approximating capabilities. Although simplified, artificial neural networks can model this learning process by adjusting the weighted connections found between neurons in the neuronal network. This effectively emulates the strengthening and weakening of the synaptic connections found in our brains. All these enable the network to learn.

2. Results and Discussions

Hausdorff and Wijsman topologies are hit-and-miss hypertopologies (Apreutesei, 2003; Beer, 1993; Gavriluț & Apreutesei, 2016; Kunze *et al*., 2012; Hu & Papageorgiou, 1997 (Ch. 1); Precupanu *et al*., 2016 (Ch. 1); Apreutesei in Precupanu *et al*., 2006 (Ch. 8); di Maio & di Naimpally, 2008).

Suppose (X,d) is an arbitrary metric space. The *Wijsman topology* τ_W on $\mathcal{P}_0(X)$ is the supremum $\tau_W^+ \cup \tau_W^-$ of the *upper Wijsman topology* τ_W^+ and the *lower Wijsman topology* τ_W^-, the family

$$\mathcal{F} = \{M \in \mathcal{P}_0(X); d(x,M) < \varepsilon\}_{\substack{x\in X\\ \varepsilon>0}} \cup \{M \in \mathcal{P}_0(X); d(x,M) > \varepsilon\}_{\substack{x\in X\\ \varepsilon>0}}$$

being a subbase for τ_W on $\mathcal{P}_0(X)$.

Remarks 1. I) If $\{M_i\}_{i\in I} \subset \mathcal{P}_0(X)$, the following statements are equivalent:

i) $M_i \xrightarrow{\tau_W} M \in \mathcal{P}_0(X)$;

ii) for every $x \in X$, $d(x,M_i) \xrightarrow{p} d(x,M)$ (pointwise convergence);

iii) $M_i \xrightarrow{\tau_W^+} M$ and $M_i \xrightarrow{\tau_W^-} M$;

II) i) $M_i \xrightarrow{\tau_W^+} M$ iff for every $x \in X$, $\liminf_i d(x,M_i) \geq d(x,M)$ (*i.e.*, for every $0 < \varepsilon < \varepsilon'$ with $S(x,\varepsilon') \cap M = \emptyset$, there is $i_0 \in I$ so that for every $i \in I$, with $i \geq i_0$, we have $S(x,\varepsilon) \cap M_i = \emptyset$).

ii) $M_i \xrightarrow{\tau_W^-} M$ iff for every $x \in X$, $\limsup_i d(x,M_i) \leq d(x,M)$ (*i.e.*, for every $D \in \tau_d$ with $D \cap M \neq \emptyset$, there is $i_0 \in I$ so that for every $i \in I$, with $i \geq i_0$ we have $D \cap M_i \neq \emptyset$).

III) (Gavriluț & Apreutesei, 2016) i) If (X,d) is a complete, separable metric space, then the family $\mathcal{P}_f(X)$ of all non-empty closed subsets of X endowed with the Wijsman topology is a Polish space (Beer, 1993). Moreover, the space $(\mathcal{P}_f(X), \tau_W)$ is Polish iff (X,d) is Polish.

ii) (X,d) is separable iff $\mathcal{P}_f(X)$ is either metrizable, first-countable or second-countable. The dependence of the Wijsman topology on the metric d is quite strong. Even if two metrics are uniformly equivalent, they may generate different Wijsman topologies.

IV) If $M \in \mathcal{P}_0(X), \{x_1, x_2, ..., x_n\} \subset X$ and $\varepsilon > 0$ are arbitrarily chosen, then τ_W^- is generated by the family $U_W^-(M, x_1, x_2, ..., x_n, \varepsilon) = \{N \in \mathcal{P}_0(X); d(x_i, N) < d(x_i, M) + \varepsilon, \forall i = \overline{1,n}\}$, while τ_W^+ is generated by the family $U_W^+(M, x_1, x_2, ..., x_n, \varepsilon) = \{N \in \mathcal{P}_0(X); d(x_i, M) < d(x_i, N) + \varepsilon, \forall i = \overline{1,n}\}$.

The *Hausdorff-Pompeiu pseudo-metric* h on $\mathcal{P}_f(X)$ is defined by

$$h(M,N) = \max\{e(M,N), e(N,M)\},$$

$\forall M, N \in \mathcal{P}_f(X)$, where $e(M,N) = \sup_{x \in M} d(x,N)$ is *the excess* of M over N and $d(x,N) = \inf_{y \in N} d(x,y)$ is the distance from x to N (with respect to the metric d).

The topology τ_H induced by the Hausdorff pseudo-metric h is called the *Hausdorff-Pompeiu hipertopology on* $\mathcal{P}_f(X)$. On $\mathcal{P}_{bf}(X)$ (the family of all nonvoid bounded closed subsets of X), h becomes a veritable metric. If, in addition, X is complete, then the same is $\mathcal{P}_f(X)$ (Hu & Papageorgiou, 1997).

Let $\mathcal{P}_k(X)$ be the family of all nonvoid compact subsets of X. We observe that $e(N,M) = h(M,N), \forall M,N \in \mathcal{P}_f(X)$, with $M \subseteq N$. Also, $e(M,N) \le e(M,P), \forall M,N,P \in \mathcal{P}_f(X)$, with $P \subseteq N$ and $e(M,P) \le e(N,P), \forall M,N,P \in \mathcal{P}_f(X)$, with $M \subseteq N$. Generally, even if $M,N \in \mathcal{P}_k(X)$, then $e(M,N) \ne e(N,M)$. If $M \in \mathcal{P}_f(X)$ and $\varepsilon > 0$, let be $S(M,\varepsilon) = \{x \in X; \exists m \in M, d(x,m) < \varepsilon\} (= \bigcup_{m \in M} \{x \in X; m \in M, d(x,m) < \varepsilon\})$,

Since $h(M,N) < \varepsilon$ iff $M \subset S(N,\varepsilon)$ and $N \subset S(M,\varepsilon)$, we get:

$$h(M,N) = \inf\{\varepsilon > 0; M \subset S(N,\varepsilon), N \subset S(M,\varepsilon)\}.$$

In other words, $\tau_H = \tau_H^+ \cup \tau_H^-$ is the supremum of the *upper Hausdorff topology* τ_H^+ (having as a base family $\{U^+(M,\varepsilon)\}_{\varepsilon>0}$, where $U^+(M,\varepsilon) = \{N \in \mathcal{P}_f(X); N \subset S(M,\varepsilon)\}$ and the *lower Hausdorff topology*

τ_H^- (having as a base the family $\{U^-(M,\varepsilon)\}_{\varepsilon>0}$, where $U^-(M,\varepsilon)=\{N\in\mathcal{P}_f(X);\ \ M\subset S(N,\varepsilon)\}$) .

Another equivalent expression of the Hausdorff distance between two sets $M,N\in\mathcal{P}_f(X)$ is:

$$h(M,N)=\sup\{|\,d(x,N)-d(x,M)\,|;x\in X\}.$$

From here, the uniform aspect of the Hausdorff topology derives - it is the topology on $\mathcal{P}_f(X)$ of uniform convergence on X of the distance functionals $x\mapsto d(x,M)$, with $M\in\mathcal{P}_f(X)$. Hausdorff topology is invariant with respect to uniformly equivalent metrics (Apreutesei, 2003).

Remarks 2. (Apreutesei, 2003; Gavriluț & Apreutesei, 2016; Precupanu *et al*., 2016 (Ch. 1); Apreutesei in Precupanu *et al*., 2006 (Ch. 8)).

i) If one replaces the pointwise convergence of Wijsman convergence by the uniform convergence (in x), then Hausdorff convergence induced by the Hausdorff pseudometric is obtained. Generally, Hausdorff topology τ_H is finer than Wijsman topology τ_W. Hausdorff and Wijsman topologies on $\mathcal{P}_f(X)$ are coincident iff (X,d) is totally bounded.

ii) If X is a real normed space, then, on the class of monotone sequences of subsets of $\mathcal{P}_k(X)$, Hausdorff and Wijsman topologies are equivalent.

iii) Hausdorff metric on $\mathcal{P}_k(X)$ is an essential tool in the study of fractals, hyperfractals, multifractals and superfractals – (Andres & Fišer, 2004; Andres & Rypka, 2012; Kunze *et al*., 2012).

The space $(\mathcal{P}_k(X),h)$ is called "the life space of fractals" (Barnsley, 1988). In a more general setting than using Hausdorff topology, a fractal approach has recently been proposed using Vietoris topology (Banakh & Novosad, 2013).

In what follows, we introduce regularity properties. An appropriate framework is the following:

Let T be a locally compact, Hausdorff space, $\mathcal{C}$ a ring of subsets of T and X a real normed space. For instance, $\mathcal{C}$ might be the Baire δ-ring $\mathcal{B}_0$ (respectively, the Baire σ-ring $\mathcal{B}_0'$) generated by the lattice of all compact subsets of T which are G_δ (*i.e.*, countable intersections of open sets) or $\mathcal{C}$ is the Borel δ-ring $\mathcal{B}$ (respectively, the Borel σ-ring $\mathcal{B}'$) generated by the lattice $\mathcal{K}$ of all compact subsets of T. Obviously, $\mathcal{B}_0 \subset \mathcal{B} \subset \mathcal{B}'$ and $\mathcal{B}_0 \subset \mathcal{B}_0'$. If T is metrisable or if it has a countable base, then any compact set $K \subset T$ is G_δ. In this case, $\mathcal{B}_0 = \mathcal{B}$ (Dinculeanu, 1964, Ch. III, p. 187), so $\mathcal{B}_0' = \mathcal{B}'$.

Regularity can be viewed as a continuity property with respect to a suitable topology on $\mathcal{P}(T)$ (Dinculeanu, 1964, Ch. III, p. 197):

In the locally compact Hausdorff space T we denote by $\mathcal{D}$ the family of all open subsets of T. and by $\mathcal{I}(K,D) = \{A \subset T / K \subset A \subset D\}$, $\forall K \in \mathcal{K}, \forall D \in \mathcal{D}$, with $K \subset D$.

We observe that $\mathcal{I}(K,D) \cap \mathcal{I}(K',D') = \mathcal{I}(K \cup K', D \cap D')$, $\forall \mathcal{I}(K,D)$, $\mathcal{I}(K',D')$. In consequence, the family $\{\mathcal{I}(K,D)\}_{\substack{K\in\mathcal{K}\\ D\in\mathcal{D}}}$ is a base of a topology $\tilde{\tau}$ on $\mathcal{P}(T)$. We denote by $\tilde{\tau}$, the topology induced on any subfamily $\mathcal{S} \subset \mathcal{P}(T)$ of subsets of T. By $\tilde{\tau}_l$ (respectively, $\tilde{\tau}_r$) we denote the topology induced on $\{\mathcal{I}(K)\}_{K\in\mathcal{K}} = \{\{A \subset T / K \subset A\}\}_{K\in\mathcal{K}}$ (respectively, $\{\mathcal{I}(D)\}_{D\in\mathcal{D}} = \{\{A \subset T / A \subset D\}\}_{D\in\mathcal{D}}$) (Dinculeanu, 1964, Ch. III, pp. 197−198).

Definition 3. A class $\mathcal{F} \subset \mathcal{P}(T)$ is *dense* in $\mathcal{P}(T)$ with respect to the topology induced by $\tilde{\tau}$ if for every $K \in \mathcal{K}$ and every $D \in \mathcal{D}$, with $K \subset D$, there is $A \in \mathcal{C}$ such that $K \subset A \subset D$.

Remarks 4. i) $\mathcal{B}_0, \mathcal{B}, \mathcal{B}_0', \mathcal{B}'$ are all dense in $\mathcal{P}(T)$ with respect to the topology induced by $\tilde{\tau}$.

ii) For every $A \in \mathcal{C}$, there always exists $D \in \mathcal{D} \cap \mathcal{C}$ so that $A \subset D$.

iii) If $\mathcal{C} = \mathcal{B}$ or $\mathcal{B}'$, then for every $A \in \mathcal{C}$, there exist $K \in \mathcal{K} \cap \mathcal{C}$ and $D \in \mathcal{D} \cap \mathcal{C}$ such that $K \subset A \subset D$.

(these statements can be easily verified since T is locally compact (Dinculeanu, 1964, Ch. III, p. 197)).

Definition 5. A set multifunction $\mu : \mathcal{C} \to \mathcal{P}_f(X)$ is *monotone* (with respect to the inclusion of sets) if $\mu(A) \subseteq \mu(B)$, for every $A, B \in \mathcal{C}$ with $A \subseteq B$.

Example 6. Suppose X is an *AL*-space (*i.e.*, a real Banach space equipped with a lattice order relation, which is compatible with the linear structure of X, such that the norm $\|\cdot\|$ on X is monotone, that is, $|x| \leq |y|$ implies $\|x\| \leq \|y\|$, for every $x, y \in X$, and also satisfying the supplementary condition $\|x + y\| = \|x\| + \|y\|$, for every $x, y \in X$, with $x, y \geq 0$).

($\mathbb{R}, L_1(\mu), l_1$ are some examples of *AL*-spaces).

Let Λ be the positive cone of X. By $[x, y]$ we mean the interval consisting of all $z \in X$ so that $x \leq z \leq y$. Suppose $m : \mathcal{C} \to \Lambda$ is an arbitrary set function, with $m(\emptyset) = 0$. We consider *the induced set multifunction* $\mu : \mathcal{C} \to \mathcal{P}_{bf}(X)$ defined for every $A \in \mathcal{C}$ by $\mu(A) = [0, m(A)]$. We observe that $h(\mu(A), \{0\}) = \sup\limits_{0 \leq x \leq m(A)} \| x \| = \| m(A) \|$, for every $A, B \in \mathcal{C}$. If, particularly, $X = \mathbb{R}$ and $m : \mathcal{C} \to \mathbb{R}_+$ is an arbitrary set function, with $m(\emptyset) = 0$, then the induced set multifunction is $\mu : \mathcal{C} \to \mathcal{P}_{bf}(\mathbb{R})$ defined for every $A \in \mathcal{C}$ by $\mu(A) = [0, m(A)] (\subset \mathbb{R})$. We observe that μ is monotone if and only if the same is m.

In what follows, let $\mu : (\mathcal{C}, \tau_1) \to (\mathcal{P}_f(X), \tau_2)$ be a monotone set multifunction, where $\tau_1 \in \{\tilde{\tau}, \tilde{\tau}_l, \tilde{\tau}_r\}$ and $\tau_2 \in \{\tau_H, \tau_W\}$ $\tau_2 = \tau_2^+ \cup \tau_2^-$, where $\tau_2^+ \in \{\tau_H^+, \tau_W^+\}$ and $\tau_2^- \in \{\tau_H^-, \tau_W^-\}$. Let $\mathcal{B}_1 \in \{ \{\mathcal{I}(K, D)\}_{\substack{K \in \mathcal{K}, \\ D \in \mathcal{D}}}, \{\mathcal{I}(K)\}_{K \in \mathcal{K}}, \{\mathcal{I}(D)\}_{D \in \mathcal{D}} \}$, be a base for τ_1 and $\mathcal{B}_2$ be a base for τ_2.

Definition 7. A set $A \in \mathcal{C}$ is (τ_2-) *regular* if $\mu : (\mathcal{C}, \tau_1) \to (\mathcal{P}_f(X), \tau_2)$ is continuous at A, *i.e.* for every $(A_i)_{i \in I}, A \subset \mathcal{C}$, with $A_i \xrightarrow{\tau_1} A$, we have $\mu(A_i) \xrightarrow{\tau_2} \mu(A)$.

When τ_1 is $\tilde{\tau}$, or, respectively $\tilde{\tau}_l$ or $\tilde{\tau}_r$, we get the notions of (τ_2-) *regularity*, (τ_2-)R_l- *regularity* (also called *inner regularity*) or (τ_2-)R_r-*regularity* (also called *outer regularity*).

The statements below easily follow:

Proposition 8. An arbitrary set $A \in \mathcal{C}$ is:

i) regular if and only if for every $\mathcal{V} \in \mathcal{B}_2$, with $\mu(A) \in \mathcal{V}$, there exist $K \in \mathcal{K} \cap \mathcal{C}, K \subset A$ and $D \in \mathcal{D} \cap \mathcal{C}, D \supset A$ so that for every $B \in \mathcal{C}$, with $K \subset B \subset D$, we have $\mu(B) \in \mathcal{V}$;

ii) R_l-regular if and only if for every $\mathcal{V} \in \mathcal{B}_2$, with $\mu(A) \in \mathcal{V}$, there exists $K \in \mathcal{K} \cap \mathcal{C}, K \subset A$ so that for every $B \in \mathcal{C}$, with $K \subset B \subset A$, we have $\mu(B) \in \mathcal{V}$;

iii) $R_r -$ regular if and only if for every $\mathcal{V} \in \mathcal{B}_2$, with $\mu(A) \in \mathcal{V},$ there exists $D \in \mathcal{D} \cap \mathcal{C}, D \supset A$ so that for every $B \in \mathcal{C},$ with $A \subset B \subset D$, we have $\mu(B) \in \mathcal{V}.$

Remark 9. I) Any $K \in \mathcal{K}$ is $R_l -$regular; any $D \in \mathcal{D}$ is $R_r -$regular.

II) For $\tau_2 = \tau_H$ or τ_W we obtain the notions of regularity from (Gavriluț & Apreutesei, 2016; Gavriluț, 2010; Gavriluț, 2012). For instance, if $\tau_2 = \tau_H$, then as a consequence of the monotonicity of μ, we have the following expressions of regularity as an approximation property (in the sense of Gavriluț, 2010):

i) *regular* if for every $\varepsilon > 0$, there are $K \in \mathcal{K} \cap \mathcal{C}, K \subset A$ and $D \in \mathcal{D} \cap \mathcal{C}, D \supset A$ so that $h(\mu(A), \mu(B)) < \varepsilon,$ for every $B \in \mathcal{C},$ with $K \subset B \subset D.$

ii) R_l *– regular* if for every $\varepsilon > 0$, there exists $K \in \mathcal{K} \cap \mathcal{C}, K \subset A$ so that $h(\mu(A), \mu(B)) =$ $e(\mu(A), \mu(B)) < \varepsilon,$ for every $B \in \mathcal{C},$ with $K \subset B \subset A.$

iii) R_r *– regular* if for every $\varepsilon > 0$, there exists $D \in \mathcal{D} \cap \mathcal{C}, D \supset A$ such that $h(\mu(A), \mu(B)) =$ $e(\mu(B), \mu(A)) < \varepsilon,$ for every $B \in \mathcal{C},$ with $A \subset B \subset D.$

III) i) μ is regular if and only if for every $\varepsilon > 0$, there are $K \in \mathcal{K} \cap \mathcal{C}, K \subset A$ and $D \in \mathcal{D} \cap \mathcal{C}, D \supset A$ so that $e(\mu(D), \mu(K)) < \varepsilon;$

ii) μ is R_l -regular if and only if for every $\varepsilon > 0$, there is $K \in \mathcal{K} \cap \mathcal{C}, K \subset A$ so that $e(\mu(A), \mu(K)) < \varepsilon;$

iii) μ is R_r -regular if and only if for every $\varepsilon > 0$, there is $D \in \mathcal{D} \cap \mathcal{C}, D \supset A$ so that $e(\mu(D), \mu(A)) < \varepsilon$.

In what follows, regularity and fractality are considered from a physical perspective. In this sense, we present some physical correspondences with hit-and-miss topologies. Several regularizations by sets of functions of ε -approximation type scale and their implications will be also provided.

The same as some physical concepts, hit-and-miss hypertopologies become consistent when considered together, although they are composed of two independent parts - the upper and the lower hypertopologies. For example, in physical terms, the non-differentiability of the curve motion of the physical object involves the simultaneous definition at any point of the curve, of two differentials (left and right). Since we cannot favor one of the two differentials, the only solution is to consider them simultaneously through a complex differential (its application, multiplied by dt, where t is an affine parameter, to the field of space coordinates implies complex velocity fields).

If we consider a fractal function $f(x)$, with $x \in [a,b]$ (one of the trajectory's equation) and the sequence of the values of the variable x:

$$x_a = x_0, x_1 = x_0 + \varepsilon, ..., x_k = x_0 + k\varepsilon, ..., x_n = x_0 + n\varepsilon = x_b. \quad (1)$$

Then by $f(x,\varepsilon)$, we denote the fractured line connecting the points

$$f(x_0), ..., f(x_k), ..., f(x_n).$$

The broken line will be considered as an approximation which is different from the one used before. We shall say that $f(x,\varepsilon)$ is an *$\varepsilon -$ approximation scale.* If we consider the $\overline{\varepsilon} -$ approximation scale $f(x,\overline{\varepsilon})$ of the same function, since $f(x)$ is similar almost everywhere, if ε and $\overline{\varepsilon}$ are small enough, then the two approximations $f(x,\varepsilon)$ and $f(x,\overline{\varepsilon})$ must lead to the same results when we study a fractal phenomenon by approximation. If we compare the two cases, then to an infinitesimal increase $d\varepsilon$ of ε, it corresponds an increase $d\overline{\varepsilon}$ of $\overline{\varepsilon}$, if the scale is dilated.

Simultaneous invariance with respect to both space-time coordinates and the resolution scale induces general Scale Relativity Theory (El-Nabulsi, 2012; El-Nabulsi, 2013). These theories are more general than Einstein's general Relativity Theory, being invariant with respect to the generalized Poincaré group (standard Poincaré group and dilatation group) (El-Nabulsi, 2012; El-Nabulsi, 2013). Basically, we discuss various physical theories built on manifolds of fractal space-time. They turn out to be reducible to one of the following classes:

i) Scale Relativity Theory (Nottale, 1993; Nottale, 2011) and its possible extensions (El Naschie *et al.*, 1995). It is assumed that the motion of microparticles takes place on continuous but non-differentiable curves. In such context, regularization works using sets of functions of $\varepsilon -$ approximation type scale;

ii) Transition in which to each point of the motion trajectory, a transfinite set is assigned (in particular, a Cantor type set (El Naschie *et al.*, 1995) $\varepsilon^{(\infty)}$ model of space-time), in order to mimic the continuous (the trans-physics). In such context, the regularization of "vague" sets by known sets works;

iii) Fractal string theories containing simultaneously relativity and trans-physics (Hawking & Penrose, 1996; Penrose, 2004). The reduction of the complex dimensions to their real part is equivalent to Scale Relativity Type theories, while reducing them to the imaginary part of their complex dimensions generates trans-physics. In such context, the simultaneous regularization by sets of functions of $\varepsilon-$approximation type scale and also by “known” sets works. The “reduction” of the complex dimensions to their real part requires the regularization by sets of functions of $\varepsilon-$approximation type scale, while the “reduction” to their imaginary part requires regularization with “known” sets. We now assume that the complex system particle moves on continuous, but non-differentiable curves (fractal curves). It is well-known that the nervous influx (through brains neuronal network) takes place on continuous but non-differentiable curves in the hydrodynamics variant of scale relativity (with arbitrary constant fractal dimension). In this way, the complex systems dynamics can be simplified and all physical phenomena involved depend not only on space-time coordinates but also on space-time scale resolution. That is why physical quantities describing the complex systems dynamics can be considered as fractal functions. Once accepted such a hypothesis, some consequences of non-differentiability by Scale Relativity Theory (SRT) are evident (Nottale, 1993; Nottale, 2011; Gavriluț & Agop, 2015a). In such perspective, some applications of non-differentiability in biological systems were recently given by Stoica *et al.*, 2015; Duceac *et al.*, 2015a; Duceac *et al.*, 2015b; Doroftei *et al.*, 2016; Nemeş *et al.*, 2015; Postolache *et al.*, 2016; Ştefan *et al.*, 2016.

i) Physical quantities that describe the complex system are fractal functions, *i.e.*, functions depending both on spatial coordinates and time as well as on the scale resolution, $\frac{\delta t}{\tau}$ (identified with $\frac{dt}{\tau}$ by substitution principle (Nottale, 1993, Nottale, 2011)). In classical physics, the physical quantities describing the dynamics of a complex system are continuous, but differentiable functions depending only on spatial coordinates and time;

ii) Two complementary representations result: the formalism of the fractal hydrodynamics (at the continuum level) on one hand, and Schrödinger’s type theory (at the discontinuum level) on the other hand. Moreover, the chaoticity, either through turbulence in the fractal hydrodynamic approach, or through stochasticization in the Schrödinger type approach, is generated only by the non-differentiability of the motion trajectories in a fractal space. We note that some consequences of the turbulence in biological systems have been recently given by Duceac, 2015c; Duceac *et al.*, 2016; Păvăleanu *et al.*, 2016, Velenciuc *et al.*, 2016; Duceac *et al.*, 2017.

3. Conclusions

A mathematical-physical perspective on fractality, regularity and several hit-and-miss hypertopologies is considered. In future works, we aim to develop a neuronal network fractal theory using Wijsman topology, since its pointwise character could characterize some properties better than Hausdorff topology does.

REFERENCES

Andres J., Fišer J., *Metric and Topological Multivalued Fractals,* Internat. J. Bifur. Chaos Appl. Sci. Engrg., **14**, *4*, 1277−1289 (2004).

Andres J., Rypka M., *Multivalued Fractals and Hyperfractals,* Internat. J. Bifur. Chaos Appl. Sci. Engrg., **22**, *1*, 1250009, 27 (2012).

Apreutesei G., *Families of Subsets and the Coincidence of Hypertopologies,* Annals of the Alexandru Ioan Cuza University – Mathematics, **XLIX**, 1−18 (2003).

Banakh T., Novosad N., *Micro and Macro Fractals Generated by Multi-Valued Dynamical Systems,* arXiv: 1304.7529v1 [math.GN], 28 April (2013).

Barnsley M., *Fractals Everywhere,* Academic Press (1988).

Beer G., *Topologies on Closed and Closed Convex Sets,* Kluwer Academic Publishers (1993).

Brown S., *Memory and Mathesis: For a Topological Approach to Psychology,* Theory, Culture and Society, **29**, *4-5*, 137−164 (2012).

di Lorenzo P., di Maio G., *The Hausdorff Metric in the Melody Space: A New Approach to Melodic Similarity,* The 9th International Conference on Music Perception and Cognition, Alma Mater Studiorum University of Bologna, August 22-26 (2006).

di Maio G., Naimpally S., *Hit-and-far-miss Hypertopologies,* Matematicki Vesnik 60, 59−78 (2008).

Dinculeanu N., *Measure Theory and Real Functions* (in Romanian), Edit. Didactică şi Pedagogică, Bucureşti (1964).

Doroftei B., Duceac L.D., Iacob D.D., Dănilă N., Volovăț S., Scripcariu V., Agop M., Aursulesei V., *The Harmonic Oscillator Problem in the Scale Relativity Theory. Its Implications in the Morphogenesis of Structures at Various Scale Resolution,* Journal of Computational and Theoretical Nanoscience, 2016 (in press).

Duceac L.D., Nica I., Gațu I., *Fractal Bacterial Growth,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 57−64, 2015a.

Duceac L.D., Nica I., Iacob D.D., *Chaos and Self-Structuring in Cellular Growth Dynamics,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 65−72, 2015b.

Duceac L.D., *Ecosocioepidemiologia-orientări moderne în medicina preventivă,* Revista Medico-Chirurgicală, Iaşi, **119**, *3*, 809−812, 2015c.

Duceac L.D., Dobre C., Păvăleanu I., Păvăleanu M., Velenciuc I., Jităreanu C., *Clinical-Epidemiological Issues of Acquired Child Methemoglobinemia,* International Journal of Medical Dentistry, **20**, *1*, 2016 (in press).

Duceac L.D., Dobre C.E., Păvăleanu I., Călin G., Nichituş S., Damir D., *Diseases Prevention by Water Defluoridation Using Hydrotalcites as Decontaminant Materials.* Revista de Chimie, **68**, *1*, 2017 (in press).

Edalat A., *Dynamical Systems*, *Measures and Fractals via Domain Theory,* Information and Computation, **120**, *1*, 32−48 (1995).

El Naschie M.S., Rösler O.E., Prigogine I., *Quantum Mechanics,* Diffusion and Chaotic Fractals, Elsevier, Oxford (1995).

El-Nabulsi A.R., *New Astrophysical Aspects from Yukawa Fractional Potential Correction to the Gravitational Potential in D Dimensions,* Indian Journal of Physics, **86**, 763−768 (2012).

El-Nabulsi A.R., *Fractional Derivatives Generalization of Einstein's Field Equations,* Indian Journal of Physics, **87**, 195−200 (2013).

Fu H., Xing Z., *Mixing Properties of Set-Valued Maps on Hyperspaces via Furstenberg Families,* Chaos, Solitons & Fractals, **45**, *4*, 439−443 (2012).

Gavriluţ A., *Regularity and Autocontinuity of Set Multifunctions,* Fuzzy Sets and Systems, **161**, 681−693 (2010).

Gavriluţ A., *Continuity Properties and Alexandroff Theorem in Vietoris Topology,* Fuzzy Sets and Systems, **194**, 76−89 (2012).

Gavriluţ A., Agop M., *A Mathematical Approach in the Study of the Dynamics of Complex Systems,* Ars Longa Publishing House (in Romanian) (2013).

Gavriluţ A., Agop M., *Approximation Theorems for Set Multifunctions in Vietoris Topology, Physical Implications of Regularity.* Iranian Journal of Fuzzy Systems, **12**, *1*, 27−42 (2015a).

Gavriluţ A., Agop M., *A Mathematical-Physical Approach on Regularity in Hit-and-Miss Hypertopologies for Fuzzy Set Multifunctions,* Mathematical Sciences, **9**, *4*, 181−188 (2015b).

Gavriluţ A., Apreutesei G., *Regularity Aspects of Non-Additive Set Multifunctions,* Fuzzy Sets and Systems (2016) (in press).

Gómez-Rueda J.L., Illanes A., Méndez H., *Dynamic Properties for the Induced Maps in the Symmetric Products,* Chaos, Solitons & Fractals, **45**, *9-10*, 1180−1187 (2012).

Hawking S., Penrose R., *The Nature of Space Time,* Princeton, Princeton University Press (1996).

Hu S., Papageorgiou N.S., *Handbook of Multivalued Analysis,* Vol. **I**, Kluwer Acad. Publ., Dordrecht (1997).

Kunze H., La Torre D., Mendivil F., Vrscay E.R., *Fractal Based Methods in Analysis,* Springer (2012).

Lewin K., Heider G.M., Heider F., *Principles of Topological Psychology,* McGraw-Hill, New York, (1936).

Li J., Li J., Yasuda M., *Approximation of Fuzzy Neural Networks by Using Lusin's Theorem,* 86−92 (2007).

Li R., *A Note on Stronger Forms of Sensitivity for Dynamical Systems,* Chaos, Solitons & Fractals, **45**, *6*, 753−758 (2012).

Liu L., Wang Y., Wei G., *Topological Entropy of Continuous Functions on Topological Spaces,* Chaos, Solitons & Fractals, **39**, *1*, 417−427 (2009).

Lu Y., Tan C. L., Huang W., Fan L., *An Approach to Word Image Mathching Based on Weighted Hausdorff Distance,* Proceedings, Sixth International Conference, 921−925, 2001.

Ma X., Hou B., Liao G., *Chaos in Hyperspace System,* Chaos, Solitons & Fractals, **40**, *2*, 653−660 (2009).

Mandelbrot B.B., *The Fractal Geometry of Nature,* W.H. Freiman, New York (1983).

Nemeş R.M., Duceac L.D., Vasincu E.G., Agop M., Postolache P., *On the Implications of the Biological Systems Fractal Morpho-Functional Structure,* U.P.B. Sci. Bull., Series A, **77**, *4*, 2015.

Nottale L., *Fractal Space-Time and Microphysics: Towards Theory of Scale Relativity,* World Scientific, Singapore (1993).

Nottale L., *Scale Relativity and Fractal Space-Time, A New Approach to Unifying Relativity and Quantum Mechanics,* Imperial College Press, London (2011).

Păvăleanu I., Gafițanu D., Popovici D., Duceac L.D., Păvăleanu M., *Treatment of Metabolic Alterations in Polycystic Ovary Syndrome,* Revista Medico-Chirurgicală, **120**, *1*, 2016 (in press).

Penrose R., *The Road to Reality: a Complete Guide to the Laws of the Universe,* London: Jonathan Cape (2004).

Postolache P., Duceac L.D., Vasincu E.G., Agop M., Nemeş R.M., *Chaos and Self-Structuring Behaviors in Lung Airways,* U.P.B. Sci. Bull., No. 6, 2016 (in press).

Precupanu A., Croitoru A., Godet-Thobie Ch., *Set-valued Integrals* (in Romanian), Iaşi, 2016 (in press).

Precupanu A., Precupanu T., Turinici M., Apreutesei Dumitriu N., Stamate C., Satco B.R., Văideanu C., Apreutesei G., Rusu D., Gavriluț A.C., Apetrii M., *Modern Directions in Multivalued Analysis and Optimization Theory* (in Romanian), Venus Publishing House, Iaşi (2006).

Sharma P., Nagar A., *Topological Dynamics on Hyperspaces,* Applied General Topology, **11**, *1*, 1−19 (2010).

Ştefan G., Duceac L.D., Gațu I., Păun V.P., Agop M., Aursulesei V., Manea L.R., Rotaru M., *Structures Morphogenesis in Complex Systems at Nanoscale,* Journal of Computational and Theretical Nanoscience, 2016 (in press).

Stoica C., Duceac L.D., Lupu D., Boicu M., Mihăileanu D., *On the Fractal Bit in Biological Systems,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 47−55 (2015).

Velenciuc I., Minea R., Duceac L.D., Vlad T., *Un Promotor Renascentist al Chirurgiei Moderne,* Revista Medico-Chirurgicală, **120**, *1*, 2016 (in press).

Wang Y., Wei G., Campbell W.H., Bourquin S., *A Framework of Induced Hyperspace Dynamical Systems Equipped with the Hit-or-Miss Topology,* Chaos, Solitons & Fractals, **41**, *4*, 1708−1717 (2009).

Wicks K.R., *Fractals and Hyperspaces,* Springer-Verlag Berlin Heidelberg (1991).

PROPRIETĂȚI DE APROXIMARE DIN PERSPECTIVĂ FIZICO-MATEMATICĂ. POSIBILE CORELAȚII CU FRACTALITATEA REȚELELOR NEURONALE

(Rezumat)

În această lucrare, sunt abordate diferite probleme referitoare la proprietatea de regularitate, văzută ca o proprietate de aproximare în unele hipertopologii de tip loveşte-şi-lasă. Punem astfel bazele pentru autosimilaritate şi fractalitate din perspectivă fizico-matematică. Întrucât în unele exemple de fractali, cum ar fi rețeaua neuronală sau sistemul circulator, proprietatea uniformă a hipertopologiei Hausdorff nu este potrivită, hipertopologia Wijsman ar putea fi preferată, putând descrie mai bine proprietățile punctuale pe care fractalii le posedă.

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 62 (66), Numărul 1, 2016
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

ON A NEW POSSIBLE CLASS OF CELLULAR NEURAL NETWORK

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Received: January 11, 2016
Accepted for publication: April 16, 2016

Abstract. In this paper, an overview concerning natural and artificial information (its generation, storage and transmission mechanisms) is intended.

Keywords: information; cellular neural network; human brain; fractal logic.

1. Introduction

Conventional digital computation methods have run into a serious speed bottleneck due to their serial nature. To overcome this problem, a new computation model, called Neural Networks, has been proposed, which is based on some aspects of neurobiology and adapted to integrated circuits. Neural Networks is a field of Artificial Intelligence, where we - by inspiration from the human brain, find data structures and algorithms for learning and classification of data. Encouraging impressive applications of neural networks have been proposed in optimization, linear and and nonlinear programming, associative

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memory, pattern recognition, computer vision (Karayiannis & Venetsanopoulos, 1993; Chow, 2007; Slavova, 2003; Ivancevic & Ivancevic, 2007).

A class of locally coupled neural networks, called Cellular Neural Networks (CNNs) were introduced (Chua & Yang, 1988a; Chua & Yang, 1988b), as a novel class of information processing systems, which possesses some of the key features of neural networks (NNs) and which has important potential applications in such areas as image processing and pattern recognition. Like a neural network, it is a large scale nonlinear analog circuit processing signals in real time. Like a cellular automata, it is made of a huge aggregate of regularly spaced circuit clone, called cells, communicating with each other directly only through its nearest neighbor. Cellular Neural Networks share the best features of these two worlds.

One of the most impressive features of artificial neural networks is their ability to learn. Although simplified, artificial neural networks can model this learning process by adjusting the weighted connections found between neurons in the network. This effectively emulates the strengthening and weakening of the synaptic connections found in our brains. This strengthening and weakening of the connections is what enables the network to learn (Karayiannis & Venetsanopoulos, 1993; Chow, 2007).

In this paper, we shall highlight the way in which a new logic (called by us *fractal logic*), can be induced in order to mimic the genetic code. Practically speaking, we discuss about a new class of cellular neural networks.

2. Results and Discussions

2.1. On Information and Wave-Corpuscle Duality

In Scale Relativity Theory, the dynamics of any physical complex system (as the human brain is) is described through variables which can be expressed through fractal functions, that is, functions which are dependent both on coordinates and on time, but also on resolution scales. Moreover, any quantity can be written as the sum between a differentiable part, *i.e.*, dependent both on coordinates and time, but also on the resolution scales (Nottale, 1993; Nottale, 2011). In such context, the differentiable part is proved to be compatible only with the predictable states of the physical system, while the fractal part is proved to be compatible only with the unpredictable states of the same physical system.

The postulate through which motions are introduced on continuous but non-differentiable curves (fractal curves) solves this problem of the straight and uniform motion, meaning that on the new fractal manifold the motion is free (on geodesics). By accepting such postulate, on the basis of the model of Scale Relativity Theory, it results that the geodesics of a fractal space-time support a double representation, a stochastic, unpredictable one, described by Schrödinger type equations and specific to the wave character, and at the same time a deterministic, predictable representation, through the fractal hydrodynamic model, which is specific to the corpuscular character. In Schrödinger's representation, only the module of the square wave function has physical significance, while in the second case we talk about average movements of some fluid particles which are submitted to a datum force, a force which is induced by the unpredictable part (non-differentiability of the motions curves) (Heisenberg, 1949).

Non-predictability, described through the non-differentiability of motion curves can be related to a Shannon-type fractal informational entropy (Weaver & Shannon, 1963; Shannon, 1948), which, based on a maximization principle, leads to an egalitarian uncertainty principle. Within this uncertainty principle, the interaction constants are specified on the basis of an Onicescu-type informational energy (Agop *et. al*, 2015; Agop *et al*., 2014; Onicescu, 1966). We mention that only the constant value of the Onicescu informational energy settles the interaction constants within the uncertainty relations. Through the maximization principle, the integrally invariant functions are simultaneously probability density and movements on constant energy curves. Practically speaking, through the principle of informational maximization, the unpredictable, wave character given by the probability density is linked to the corpuscle character given by the energy. The unpredictable part must be directly correlated to non-differentiability and it is manifested through the existence of a potential, also called fractal potential. The principle of maximization of the informational energy gives a concrete form to the force field. As a result, the informational energy not only stores and transmits the information through interaction, but also connects it directly to the deterministic part through interaction. So, practically speaking, the owner of all "mysteries" is the fractal potential, which imposes the intelligent, fractal environment and the informational energy which gives strength.

As above-specified, on the basis of the non-predictable component, one can define a fractal entropy in Shannon's sense and, starting from here, a fractal informational energy in the sense of Onicescu. By using a maximization principle of fractal entropy in Shannon's sense, one can demonstrate that, if the fractal informational energy in Onicescu's sense is constant, then the ratio between the corpuscle energy and the frequency of the associated wave is a constant at any scale resolution. Quantification at any scale resolution is thus induced. Particularly, for motions on Peano curves (of fractal dimension $D_F = 2$) and for Compton scale, the usual quantification is obtained, a situation which corresponds to quantum computers class.

2.2. Computational Models Inspired by Human Brain Neuronal Networks

Complex systems theory enables a holistic approach concerning brain structure and human brain functions. Many phenomena with complex patterns and structures are widely observed in the brain. These phenomena are some manifestations of a multidisciplinary paradigm called emergence or complexity. They share a common unifying principle of dynamic arrays, namely, interconnections of a sufficiently large number of simple dynamic units can exhibit extremely complex and self-organizing behaviors (Jackson, 1992; Stonier, 1990; Yockey, 1992; Wicken, 1987).

Artificial neural networks are inspired by the biological nervous system, in particular, the human brain. One of the most interesting characteristics of the human brain is its ability to learn. We should note that our understanding of how exactly the brain does this is still very primitive, although we do still have a basic understanding of the process. It is believed that during the learning process the brain's neural structure is altered, increasing or decreasing the strength of its synaptic connections depending on their activity. This is why more relevant information is easier to recall than information that has not been recalled for a long time. More relevant information will have stronger synaptic connections and less relevant information will gradually have its synaptic connections weaken, making it harder to recall.

The increased availability of computing power has not only made many new applications possible but has also created the desire to perform cognitive tasks which are easily carried out by the human brain. It become obvious that new types of algorithms and/or circuits were necessary to cope with such tasks. Inspiration has been sought from the functioning of the human brain, which led to the artificial neural network approach. One way of looking at neural networks is to consider them to be arrays of nonlinear dynamical systems that interact with each other.

The highly interdisciplinary nature of the research in CNNs makes it very difficult for a newcomer to enter this important and fascinating area of modern science. This permits the mathematical modeling and analysis of networks of neurons from the viewpoint of dynamical complex systems.

Under certain restrictions, cellular neural networks (CNNs) come very close to some Hamiltonian systems, they are potentially useful for simulating or realizing such systems. They show how to map two one-dimensional nonlinear lattices, the Fermi-Pasta-Ulam lattice (1965) (Chua & Yang, 1988a; Chua & Yang, 1988b) and the Toda lattice (Toda, 1981; Toda, 1983), onto a CNN. For the Toda lattice, they show what happens if the signals are driven beyond the linear region of the piecewise-linear output function. Though the system is no longer Hamiltonian, numerical experiments reveal the existence of soliton solutions for special initial conditions. This interesting phenomenon is due to a special symmetry in the CNN system of ordinary differential equations. If one is only interested in the signals associated with Hamiltonian systems, and not in conserving the energy in individual circuit elements (nonlinear inductors and capacitors), then such systems can be built as analog circuits, which implement some signal flow graphs.

Considering that the nervous impulse transmission through brain neuronal network is achieved on continuous but non-differentiable curves, in Scale Relativity Hydrodynamic variant (with constant arbitrary fractal dimension), "information" (through states density) is "stored" and "transmitted" by spatial-temporal cnoidal modes. These modes can be assimilated to a one-dimensional Toda network and by mapping, to a cellular neural network. Cnoidal modes double periodicity generates differentiable-non-differentiable Toda pair and further by mapping, differentiable-non-differentiable cellular neural network pair. In such conjecture, a topological method can be applied since the admissible number of fractal kinks (cnoidal modes degeneration for the case of non-linear "functionality" regime) is determined by the topological properties of the symmetry group associated to the differential equation of these modes. In this way, two distinct states are thus established for each of the components of the pair. This induces in total four distinct states, as stipulated by the human genetic code. These four distinct states could substantiate a new type of logic, which we call *fractal logic*.

Pairs' coherence specifies that information's "storage" and "transmission" is achieved by specific algebraic languages.

Unlike the electronic calculator which has a hard structure divided by artificial algorithms, the spectral component corresponding to the hardware particles, has the same artificial behavior, without having the fractality of the natural development. As a result, there is no consistency between the cellular network of the substance and the spectral one. The development of the neural network is performed according to fractal criteria, the same as for all the other parts and systems of the human body. Consequently, the spectral field created by the undulation of the particles of the neural network is consistent, allowing the information to be processed inside the neural network and also in the spectral field (Hilbert space), where there are the a-spatial and a-temporal components which enable the memory and also the complex component that allows the possibility of multidimensional processing which can explain the superior psychic processes, the conceptualization, the semantics, the abstracting.

Therefore, at any scale there are the two types of realities that coexist, the differential one and the non-differential one, highlighted by the hydrodynamic and stochastic theories. Another major difference between the electronic calculator and the human brain is the analogical feature of psychic processing, unlike the digital signal processing. Analog signal processing is enhanced by the topology configuration character of the processing, being not only a numeric processing, but also one determined by the geometric topology.

2.3. Information as an Expression of Topological Transformations

Related to the above statements from the previous section, "topology" proves to play a fundamental role in the definition of a fractal logic, so, the paradoxes highlighted by quantum mechanics in the first half of the 20th century include, apart from the uncertainty relations (Heisenberg, 1949), a strange involvement of the observer in developing quantum phenomena. The Copenhagen school avoided to attribute a significance to these observations, but today they must be researched also from a philosophical and methodological aspect. Anyway, these facts suggest that the splitting into subjective and objective information is artificial and that they should be regarded as aspects of the same phenomenon. In order to uphold this idea, we must take into consideration another paradox of quantum mechanics, which is just as exciting and linked to the entanglement phenomenon, which, as a result of repeated experiments, highlighted a reality which is hard to infer, that is, that all the particles which interacted at a certain point remain connected.

All these paradoxes that quantum mechanics imposed, along with the wave-corpuscle duality, determined a new approach in physics, mathematics and in the scientific approach in general. If during the 20th century it was studied from the elementary particles' point of view, of the wave component from the spectral viewpoint and materially under the form of substance and energy, the information was not treated at its true value, according to the role it has in quantum mechanics. The information technology era, as well as the complex systems theory, with the chaotic aspects in which information has a potential character, but which explains the dynamic evolution patterns of the system which is highlighted in the phase space, have imposed the comeback on the role of information at quantum level. An analysis of the particle behavior in the wave-corpuscle duality can be regarded from the fractal space-time perspective, with the unpredictable and non-linear evolution, allowing that, on the basis of Shannon's information theory (Weaver & Shannon, 1963; Shannon, 1948) we connect it to entropy and further to informational energy in the sense of Onicescu. There still remains an essential question: where can we search for and find the information in this quantum dynamics. It must be present both in the wave structure and in the particle properties. This connection cannot be made otherwise than in the phase component of the wave, which is to be found in the spinning of the particle and which allows for the transfer of information from the spectral reality to the corpuscle one, as the Fourier transform demonstrates. The phase is given by the magnetic component of the electromagnetic field and represents the unpredictable, potential part, described by the complex function of Schrödinger's wave formula, as these characteristics can be explained both through the fractal theory and through the topological transformations supported by the phase from the electromagnetic wave, respectively by the spin from the particle description.

Complex numbers proved to be the most suitable in describing the rotation movement around its own axis, but this model is a dynamical one, which supports transformations at the level of topological dimension through successive passage from the topological dimension 0 (of the point) to the topological dimension 1 (of the line) etc. The dimensional dynamics, from 0 to infinite, which is realized in our reality up to 3 dimensions, can be performed multi-dimensionally in the complex field of psychic reality (through the fractal potential).

Thus, an infinite dimensional complex space is organized, which explains the difficulty of highlighting the informational component. The successive passage through the Euclidean, fractal and topological dimension determines a quantitative but also qualitative dynamics of energy. The moment this qualitative diversity is expressed, is given by the moment of topological transformations at every level. This practically-unlimited diversity provides also quality along with quantity to energy in its dynamics. From the complex systems theory perspective, we can find in the above-described phenomena the main characteristics specific to complex systems: non-linear dynamics, fractal geometry, with a potential latent informational energy, along with a dynamics of a practically-infinite diversity, obtained through topological transformations within the complex space of the phase. In this sense, Stoica *et al*., 2015; Duceac *et al*., 2015a; Duceac *et al*., 2015b; Doroftei *et al*., 2016; Nemeş *et al*., 2015; Postolache *et al*., 2016; Ştefan *et al*., 2016 presented consequences of topological transformations in biological systems.

The topological transformations are not dependent on scale; they have the same qualitative information, no matter what the reality level is, which makes the information ubiquitous, just as the substance and energy both at microcosm level and at macrocosm level.

3. Conclusions

Information represents codified energy which is expressed under the form of patterns, structure patterns, initiated by attractors which activate in the phase space, between the chaotic and the structured part. The information is stored in the spectral space and expresses the patterns in the structure of atoms, molecules, macromolecules and cells. It has a potential existence which is expressed through substance and energy in certain conditions of local coherence. Information is to be found in the complex space of the spectral field of the wave phase. As a result, this complex space is everywhere and at any level of reality. The space of physical reality is intertwined with the complex space generated by electromagnetic waves. From the mathematical viewpoint, this space is infinite dimensional as it may contain the whole information in the Universe. At quantum level, this overlapping is done through collapsing the wave in the complex space of the wave phase level and it is transmitted to the complex space of the spin rotation, thus transferring the whole information. This phenomenon is specific to the reality from the level of the whole cognoscible Universe, as electromagnetic waves exist anywhere and also at every level of the reality, including the human brain. Thus, the brain has access both to the real and the complex space, as it manages to become aware both of the physical reality through analyzers and instruments, which are the prolongation of analyzers, but at the same time to become aware of the complex reality which represents a source of creation, inspiration and knowledge, beyond what is offered to us through the perception of analyzers.

In this sense, various dynamics imposed by means of information on the biological systems were recently given by Duceac, 2015; Duceac *et al*., 2016; Păvăleanu *et al*., 2016; Velenciuc *et al*., 2016; Duceac *et al*., 2017.

REFERENCES

Agop M., Gavriluț A., Crumpei G., Doroftei B., *Informational Non-Differentiable Entropy and Uncertainty Relations in Complex Systems,* Entropy, **16**, *11*, 6042−6058 (2014).

Agop M., Gavriluț A., Rezuş E., *Implications of Onicescu's Informational Energy in Some Fundamental Physical Models,* International Journal of Modern Physics B, **29**, *0* (2015).

Chow T.W.S., *Neural Networks and Computing,* Learning Algorithms and Applications (Series in Electrical and Computer Engineering) (2007).

Chua L.O., Yang L., *Cellular Neural Networks: Applications,* IEEE Transactions on Circuits and Systems, **35**, *10*, 1273−1290 (1988).

Chua L.O., Yang L., *Cellular Neural Networks: Theory,* IEEE Transactions on Circuits and Systems, **35**, *10*, 1257−1272 (1988).

Doroftei B., Duceac L.D., Iacob D.D., Dănilă N., Volovăț S., Scripcariu V., Agop M., Aursulesei V., *The Harmonic Oscillator Problem in the Scale Relativity Theory. Its Implications in the Morphogenesis of Structures at Various Scale Resolution,* Journal of Computational and Theoretical Nanoscience, 2016 (in press).

Duceac L.D., *Ecosocioepidemiologia-orientări moderne în medicina preventivă,* Revista Medico-Chirurgicală, Iaşi, **119**, *3*, 809−812, 2015.

Duceac L.D., Nica I., Gațu I., *Fractal Bacterial Growth,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 57−64, 2015a.

Duceac L.D., Nica I., Iacob D.D., *Chaos and Self-Structuring in Cellular Growth Dynamics,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 65−72, 2015b.

Duceac L.D., Dobre C., Păvăleanu I., Păvăleanu M., Velenciuc I., Jităreanu C., *Clinical-Epidemiological Issues of Acquired Child Methemoglobinemia,* International Journal of Medical Dentistry, **20**, *1*, 2016 (in press).

Duceac L.D., Dobre C.E., Păvăleanu I., Călin G., Nichituş S., Damir D., *Diseases Prevention by Water Defluoridation Using Hydrotalcites as Decontaminant Materials,* Revista de Chimie, **68**, *1*, 2017 (in press).

Heisenberg W., *The Physical Principles of the Quantum Theory,* Courier Dover Publications (1949).

Ivancevic V.G., Ivancevic T.T., *Neuro-Fuzzy Associative Machinery for Comprehensive Brain and Cognition Modeling,* Studies in Computational Intelligence, **45**, Springer (2007).

Jackson E.A., *Perspectives on Nonlinear Dynamics,* Cambridge University Press, Cambridge (1992).

Karayiannis N.B., Venetsanopoulos A.N., *Artificial Neural Networks,* Learning Algorithms, Performance Evaluation, and Applications, The Springer International Series in Engineering and Computer Science, **209** (1993).

Nemeş R.M., Duceac L.D., Vasincu E.G., Agop M., Postolache P., *On the Implications of the Biological Systems Fractal Morpho-Functional Structure,* U.P.B. Sci. Bull., Series A, **77**, *4*, 263−272, 2015.

Nottale L., *Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity,* World Scientific, Singapore (1993).

Nottale L., *Scale Relativity and Fractal Space-Time: A New Approach to Unifying Relativity and Quantum Mechanics,* Imperial College Press, London (2011).

Onicescu O., *Enérgie informationnelle,* C.R. Acad. Sci. Paris A, **263**, 841−842 (1966).

Păvăleanu I., Gafițanu D., Popovici D., Duceac L.D., Păvăleanu M., *Treatment of Metabolic Alterations in Polycystic Ovary Syndrome,* Revista Medico-Chirurgicală, **120**, *1*, 2016 (in press).

Postolache P., Duceac L.D., Vasincu E.G., Agop M., Nemeş R.M., *Chaos and Self-Structuring Behaviors in Lung Airways,* U.P.B. Sci. Bull., *6*, 2016 (in press).

Shannon E., *Mathematical Theory of Communication,* The Bell System Technical Journal, **27**, 379−423 (1948).

Slavova A., *Cellular Neural Networks: Dynamics and Modeling,* Mathematical Modelling: Theory and Applications, **16**, Springer (2003).

Ştefan G., Duceac L.D., Gațu I., Păun V.P., Agop M., Aursulesei V., Manea L.R., Rotaru M., *Structures Morphogenesis in Complex Systems at Nanoscale,* Journal of Computational and Theretical Nanoscience, 2016 (in press).

Stoica C., Duceac L.D., Lupu D., Boicu M., Mihăileanu D., *On the Fractal Bit in Biological Systems,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 47−55, 2015.

Stonier T., *Information and the Internal Structure of the Universe,* Springer Verlag, London, 155 (1990).

Toda M., *Nonlinear Lattice and Soliton Theory,* IEEE Trans. CAS, **30**, 542−554 (1983).

Toda M., *Theory of Nonlinear Lattices,* New York: Springer Verlag (1981).

Velenciuc I., Minea R., Duceac L.D., Vlad T., *Un promotor renascentist al chirurgiei moderne,* Revista Medico-Chirurgicală, **120**, *1*, 2016 (in press).

Weaver W., Shannon C.E., *The Mathematical Theory of Communication,* Univ. of Illinois Press (1963).

Wicken J.S., *Evolution, Thermodynamics and Information*: *Extending the Darwinian Program,* Oxford University Press, May (1987).

Yockey H., *Information Theory and Molecular Biology,* Cambridge University Press (1992).

ASUPRA UNEI POSIBILE NOI CLASE DE REȚELE
CELULARE NEURALE

(Rezumat)

În această lucrare, realizăm o prezentare de ansamblu asupra subiectului informației naturale şi artificiale (sunt vizate mecanismele de generare, stocare şi transmitere).

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 62 (66), Numărul 1, 2016
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

BEYOND THE QUANTUM PARADIGM, AN EPISTEMIOLOGICAL APPROCH

BY

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Received: January 8, 2016
Accepted for publication: April 12, 2016

Abstract. The quantum paradigm as a discreet method of describing the structure of the material world, fundamentally based on quantum mechanics, is in contradiction with the electromagnetic theory of light. Promoting a mechanical vision of subatomic phenomena has been a step backwards in knowledge by going back to a convoluted theory regarding the corpuscular character of light which was extended over the structure of the material world.

The complexity of the processuality of the material world as a unity between the discreet corpuscular character and the continuous electromagnetic manifestations reveal a new theory of the physical reality, in a complex epistemic vision, in which the two ontological entities are not mutually exclusive, but rather they coexist in a unified theoretical system explained by a new electrodynamic approach of the material world which reveals a spatiotemporal universe is both knowable and predictable.

Keywords: paradigm; quantum mechanics; processuality; electrodynamic.

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1. Introduction

Scientific knowledge in its development has been, and still is, marked by tension generated by the theoretical contradictions that have appeared between the various methodologies in the approach to theory, that sometimes get a conflictual character.

The current crisis in theoretical physics has been signalled by Karl Popper in his work "Quantum theory and the schism in physics": "One was to shed some light on the significance of the present crisis in physical theory: the rejection of the Faraday-Einstein-Schrodinger programme has left us without any unifying picture, without a theory of change, without a general cosmology. Instead of a problem situation within a research programme, or relative to a research programme, our fundamental problem situation arises from a schism in physics-from a clash between two research programmes, neither of which seems to be doing its job" (Popper, 1982, p. 173). This crisis does not represent just a collision between two research programs, it is not a subjective crisis. The profound significance of this crisis refers to the agnostic and indeterministic character promoted by quantum theory in the causal determinism of classical physics which looks for a unified solution for both the microcosm and the macrocosm.

In the same book Karl Popper reveals the way in which the attempt to solve this crisis was tried, and how it failed: "Einstein's and Schrodinger's inspiring programme has been attacked by quantum theorists and, according to the judgement of most physicists, has been successfully killed. But those who attacked it have made hardly any attempt to replace it by a similarly powerful programme" (Popper, 1982, p. 173).

The crisis in the current physics reveals the contradiction generated by the discreet mechanical character of quantum theory by eluding electromagnetism as a theoretical basis, without acknowledging its profound significance - the inoperant character of quantum mechanics in the processual description of the structure of the material world.

The major crisis that has manifested between the quantum and classical approaches by forcefully imposing a reductionist mechanical interpretation was in detriment to a complex processual approach in accord with the classical fundament in theoretical physics. The result of describing the structure of the material world from a quantum perspective reveals to us a microcosm of undetermined probabilism.

From the perspective of classical physics, quantum theory brings a crisis of blurring the dichotomy, the distinction regarding the corpuscular character of particles and the continuous character of electromagnetic phenomena as a fundament of their processual aggregation. The removal of this dichotomy by quantum theory has represented an enormous step backwards in knowledge and has created the confusion between the similarity or identity of the two fundamental ontological entities of matter - waves and particles.

As was presented in the published book *The End of Quantum Theory*: “The founders of the quantum theory of substance’s structure have focused, in their theoretical approach, on the elaboration of the theory that would explain the structure of the material world in the context of insufficient development of the scientific knowledge, so that they had not all the theoretical instruments necessary to solve such a complex issue at that particular moment” (Hodorogea, 2008, p. 8).

The method of solving a crisis in knowledge, like the one which exists now in quantum physics has been explained by Thomas Kuhn: “The transition from a paradigm in crisis to a new one from which a new tradition of normal science can emerge is far from a cumulative process, one achieved by an articulation or extension of the old paradigm. Rather it is a reconstruction of the field from new fundamentals, a reconstruction that changes some of the field's most elementary theoretical generalizations as well as many of its paradigm methods and applications. During the transition period there will be a large but never complete overlap between the problems that can be solved by the old and by the new paradigm. But there will also be a decisive difference in the modes of solution. When the transition is complete, the profession will have changed its view of the field, its methods, and its goals” (Kuhn, 1996, p. 85). The radical solution presented by Thomas Kuhn is to fundamentally rewrite quantum theory and change the set of theoretical principles.

The complexity of the processuality of the material world as we understand it, as a unity between the corpuscular character and the electromagnetic manifestations in the diversity of their spatiotemporal manifestation reveals a new theory for the physical reality, in a complex epistemic vision, in which the two ontological entities are not mutually exclusive, but rather they coexist in a unified theoretical system which reveals a spatiotemporal universe which is both knowable and predictable.

The current crisis in theoretical physics is manifested in the conceptual basis of the character of the material world in regards to the relationship between continuous and discreet. Is the material world continuous or discreet?

In his work *Matter and Mind a Philosophical Inquiry,* Mario Bunge shows that: “Electromagnetic field theory, born in the 1830s, changed not only the ontology of classical physics but also its methodology. Indeed, consider the problem of finding data about two very different universes: Newton’s, constituted by corpuscles, and Faraday’s, filled with fields” (Bunge, 2010, p.180). Many physicists and epistemologists consider the universe of the material world to have a corpuscular nature, and thus discreet. In their mechanical vision they dismiss the gravitational field as a continuous manifestation of matter.

We need to understand the universe in its integrity, as a symbiotic relationship between the corpuscular existence of matter and the continuous existence of the electromagnetic and gravitational fields.

Einstein's mechanical view, which on the basis of which quantum theory was built, when referring to the structure of matter is expressed and assumed to be as a subjective theoretical option with a discreet nature.“In accordance with the assumption to be considered here, the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units” (Einstein, 1965, p.1). This assumption is the result of the subjective option generated by the theoretical principles which are the basis of quantum theory. The mechanical discreet option in describing the world promoted by the creators of quantum theory collides with the wave character of light, which is denied and excluded from the theoretical space of knowledge without a fundamentally scientific reason.

The theories of James Clerk Maxwell regarding the manifestation of the electric and magnetic fields as physical measures of the state of a continuous matter have not been completely understood, and haven't found their place in a satisfying all encompassing theory that describes the fundamental structure of matter. This was due in part to the ideas about an ether, and a tendency for a discreet mechanical interpretation of the physical phenomena, which pays tribute to Newtonian mechanics and the fact that mechanical movement, under all of its forms, is intuitively easier to understand and accept compared to the electromagnetism of electromagnetic waves.

2. The Quantum Paradigm

A change in paradigm is not always a revolution in knowledge, it can also be an involution. Tomas Kuhn defines the paradigm as: “universally recognized scientific achievements that, for a time, provide model problems and solutions for a community of practitioners” (Kuhn, 1996, p. 10).

Paradigm knowledge is a summary of scientific knowledge focused on the major changes in methodology that generate new solutions. It removes from scientific knowledge the construction and the finality of the theoretical description. Making the theoretical substantiation of principles to a secondary concern represents a limitation of knowledge in its entirety, and concentrates the focus on its finality. A paradigm is an abstract concept in scientific knowledge, adopted as essential syntax of scientific theories.

The validity of a theory is not based on its acceptance, which is a psychological option regarding a theory of a group of researchers or the entire scientific community. The revolutions in knowledge defined by Kuhn as paradigms represent inflexion points on the road towards knowledge.

The paradigm definition of the bivalent nature of the theory as both method and solution introduces a dilemma between the theoretic model and the theoretic solution. *Kuhn's paradigm* is in fact the scientific theory without its fundamental principles. Removing the principles from the set of ideas that are the basis of knowledge blurs the internal coherency that is specific to scientific knowledge. The paradigm defined knowledge promoted by Thomas Kuhn does not analyze complex systems and doesn't notice the fact that the theories that propose to describe complex systems need a different type of approach. Contrary to Kuhn we consider that the scientific revolutions are based on a new set of principles that include both scientific revolution and theoretical development.

When he analyzes quantum mechanics Thomas Kuhn does not notice, in his book *The Structure of Scientific Revolutions* the complexity of this theory and the theoretical incompatibility given by the existence of the two paradigms referring to the particle and wave characters of light in the frame of the same theory generated by the wave-particle dualism. In his analysis he does not notice the fact that the theories that are supposed to describe complexities are based sometimes on subsidiary paradigms of other theories.

Thomas Kuhn does not mention the wave-particle dualism. He does refer to optical physics and its paradigms which he analyses as a problem external to quantum physics: “These transformations of the paradigms of physical optics are scientific revolutions, and the successive transition from one paradigm to another via revolution is the usual developmental pattern of mature science. It is not, however, the pattern characteristic of the period before Newton's work, and that is the contrast that concerns us here” (Kuhn, 1996, p.12).

In the interpretation of the structure of scientific revolutions promoted by Thomas Kuhn based on paradigms, symbolic concepts in the evolution of knowledge, the unity and solidity of science seems to lose its meaning. “If normal science is so rigid and if scientific communities are so close-knit as the preceding discussion has implied, how can a change of paradigm ever affect only a small subgroup? What has been said so far may have seemed to imply that normal science is a single monolithic and unified enterprise that must stand or fall with any one of its paradigms as well as with all of them together. But science is obviously seldom or never like that” (Kuhn, 1996, p.49).

The monolithic unity of scientific knowledge must be comprehended in the theoretical principles that are the basis of knowledge which must include the theoretical solutions.

3. The Critique of Quantum Theory

We ask ourselves the obvious question: how did we end up with the corpuscular interpretation of light seeing as the electromagnetic theory had confirmed its validity and had been unanimously accepted as Einstein himself said in his article “Concerning an Heuristic Point of View Toward the Emission and Transformation of Light” (Einstein, 1965, p.1) from 1905, article which can be considered the starting point for the author which would become one of the most fascinating personalities of the scientific community, and who left his mark on the development of modern physics by attempting to explain the field-matter interaction. "The wave theory of light, which operates with continuous spatial functions, has worked well in the representation of purely optical phenomena and will probably never be replaced by another theory. It should be kept in mind, however, that the optical observations refer to time averages rather than instantaneous values. In spite of the complete experimental confirmation of the theory as applied to diffraction, reflection, refraction, dispersion, etc., it is still conceivable that the theory of light which operates with continuous spatial functions may lead to contradictions with experience when it is applied to the phenomena of emission and transformation of light" (Einstein, 1965, p.1).

From this quote we can see that Einstein does not dispute the wave theory of light as it has been confirmed by the diffraction, reflection, refraction, and dispersion experiments but suggests a contradiction between the continuous character of light as a manifestation of electromagnetic radiation and the proposed discreet character of the structure of matter. The error in the interpretation results from not understanding correctly the structure of matter, more precisely the continuous character of the internal energy of the structure of matter which gives us the laws by which the structure is achieved. The erroneously simplistic vision and the logical extrapolation - the discreet character of mass, which imposes by necessity a discreet character for the internal energy of matter - has contributed to the development of quantum theory on flawed fundamental principles.

Albert Einstein expresses without doubt a simplistic, discreet, mechanical vision of electromagnetic phenomena "It seems to me that the observations associated with blackbody radiation, fluorescence, the production of cathode rays by ultraviolet light, and other related phenomena connected with the emission or transformation of light are more readily understood if one assumes that the energy of light is discontinuously distributed in space" (Einstein, 1965, p.1). There are many physicists that consider that the majority of phenomena in physics can be explained and reduced to mechanical models. This idea explains the apparently illogical return to a mechanical model in describing the structure of matter. A simplistic solution was adopted, one which is inadequate for the complexity of the material world that we want to describe in a scientific manner.

In my opinion, presented in the book The End of Quantum Theory as well as other articles, I have shown the artificial character of quantum theory as a formal construct of a virtual reality based on principles adapted from the theoretical system.

The theoretical principles substantiate theories, they are not adjustable parameters which we can modify in our cognitive process in order to insure internal coherency. Our theoretical constructs are not a substitute for the real world, or mental projection of reality mediated by scientific knowledge do not become reality itself.

The excessively mathematic description does not add to the knowledge and does not legitimize it. To some extent it limits the access to those that are unfamiliar with the mathematical description, and in actuality it forcefully imposes an inadequate theoretical set of formulas.

The unilaterality and simplism of thought of Thomas Kuhn is given by his strictly methodological approach to the analysis of scientific knowledge. The grand scientific revolutions are based on the restructuring of theoretical principles which implicitly generate specific methods of theoretical development and "fundamental paradigms". The quantum paradigm obscures the fragility of the theoretical fundaments of quantum mechanics, referring to its atemporal and indeterministically agnostic character.

Mario Bunge is situated, as other epistemologists are, on mechanical principles that originate in the antique way of thinking that has been perpetuated until our times under various forms."All material things are either elementary, such electrons and quarks, or systems of such. In other words, things do not come in arbitrary amounts, and they cannot be divided into arbitrary parts. Thus, the ancient atomists were basically right" (Bunge, 2010, p.41).

The mechanical vision is a temptation, easily accessible to our minds as well as an obstacle in the understanding of the complexity of the material world as a wave structure in its entirety. Electromagnetic energy is not arbitrary. It has a spatiotemporal description completely defined by intensity, frequency, phase, polarity and spatial temporal repartition.

Mario Bunge's interpretation in regards to energy is a simplistic one considering energy as a property of matter and not matter itself. "Moreover, some properties too are quantized. For example, the energy of an atom in a stationary state cannot take arbitrary values: it can only be in one in an infinite denumerable set" (Bunge, 2010, p. 56).

Energy in the atomic architecture represents the structure, it is the bond that links elementary particles, it configures and determines the movement of particles on perfectly defined trajectories and defines the properties of the structure of matter.

The atomic structure in general and the structure of the material world in its entirety mean energy, mean the dynamics of elementary particles. On this point the atomic structure and the celestial structure find themselves in the same processual dynamic description. Corporality in the structure of the universe of the material world from the microcosms to the macrocosms, related to the space that it occupies is tiny. The paradigm shift in the description of the material world is represented by accepting energy under all its forms of manifestation, especially of electromagnetic energy and a defining element of the processuality of material existence.

If we interpret quantum theory as the ultimate report to the real world that presumably it describes, as a general trait we can say that the probabilistic interpretation, the quantum indeterminism is not found in the precisely defined properties of all substances.

Jon Bell tells us in his book "*Speakable and unspeakable in quantum mechanics*" about the obscure fundamental character of quantum mechanics: "As for technical mistakes, our theorists do not make them. And they see at once what is important and what is detail. So it is another feature of contemporary progress …. This progress is made in spite of the fundamental obscurity in quantum mechanics. Our theorists stride through that obscurity unimpeded ... sleepwalking?

The progress so made is immensely impressive. If it is made by sleepwalkers, is it wise to shout 'wake up'? I am not sure that it is. So I speak now in a very low voice" (Bell, 1987, p.170).

It must be shown that Einstein, even in 1906, has critiqued the way in which Plank has constructed, from a formal point of view, his constant by considering it inconsistent from a physical point of view. Still in subsequent situations where he had to take a position on this he said that this inconsistency does not represent a reason to dismiss quantum theory in its entirety. I believe that from a subjective point of view we must see Einstein's position, since as he was involved in finding a theoretical solution for the atomic structure he might have been following: "*Se non è vero, è bene trovato*" (*If it's not true, at least it's well invented*) and, in the absence of another possibility, has accepted without criticism the only theoretical solution available at that time.

Considering all of this, from the point of view of a severe critique, we can conclude that Plank's constant is theoretically incompatible from the point of view of the formal development by using in its development two mutually exclusive theories (one based on the continuous character of energy and one on its discreet character) which demonstrates the existence of an internal contradiction which is unacceptable and points out the lack of theoretical coherency in the way the fundamental constant of quantum mechanics was found.

This is why we consider as founded Paul Marmet's criticism of the ensemble of modern physics: "The contradictions found in modern science are so *absurd* that most physicists assume that somebody must certainly have solved them long ago. The degree of indifference of most physicists about these contradictions is phenomenal" (Marmet, 1993, p.11).

Considerable efforts have been made to sustain and present quantum theory as the only and irreplaceable theory of the atomic structure so it is very difficult, or maybe impossible for those that are involved in education, and not only them, to recognize that their scientific convictions regarding quantum theory are absurd and must be replaced.

In our opinion a theory based on obscure principles assumed to be indeterministic and atemporal cannot represent a starting point for a credibly valid theoretical system. It maintains in its development the indeterministic character and despite the apparent progress quantum mechanics must be replaced by a deterministic theory appropriate for the classical spirit, compatible with the wave character of light. Progress in quantum theory must be understood as one of circumstance and short termed compared to the old existing theories regarding the representation on the microcosm. In its entirety scientific knowledge is a theory based on indeterministic principles and at the same time a convoluted theory since it comes back to a corpuscular representation of the world compared to the wave representation.

4. The Necessity for an Electromagnetic Foundation Regarding the Structure of the Material World

Quantum theory is in essence a theory of states of discreet energy oscillations regarding the structure of the material world, in contradiction with the electromagnetic theory of the world which represented a continuous description of electromagnetic radiation and the basis for the electrodynamic theory of matter as a continuous theory. In the conceptual development of quantum theory the next explanatory step after the discreet definition of the energy states of the structure of matter is done by discussing the photoelectric effect as a discreet fundamental basis for quantum theory and as the explanation for the interaction between matter and the electromagnetic radiation (light) that has s an effect electron emission. This narrow mechanical vision of the phenomena has represented a regress in knowledge by returning to a convoluted theory regarding the corpuscular character of light. The electromagnetic interaction is viewed as a collision and the long distance interaction with the gravitational field is ignored without denying the long distance universal attraction and implicitly the continuous character of universal attraction.

The theory that we propose regards a wave structure of matter which becomes theoretically compatible with the wave character of light and consequently a representation where the interaction between electromagnetic waves and matter is possible.

The means for theoretical expression at the beginning of the 20th century were insufficient for a spatiotemporal description of the atomic structure. Since they couldn't develop a model for the structure of matter which could be theoretically compatible with the wave character of light, quantum theory has taken a simplistic mechanical approach and applied this model to both the atomic structure and light. Because it could not generalize the corpuscular character of light and couldn't deny its electromagnetic structure it took a dualist solution. Light, in the process of propagation, has a wave character and in the moment of interaction with matter it takes a corpuscular character. They can't explain this transformation. The conceptual fundamental problem referring to the dualism is that in a unified theoretical system there cannot be two theories that are mutually exclusive. It is difficult to understand how we returned to the corpuscular conception of light after wave theory has been completely explained and phenomena like interference and diffraction have been experimentally validated.

Quantum theory explains the passing of an electron from one orbital to another as a result of the action of light over matter. This concept of the photon-matter interaction has been taken and used on a large scale to explain chemical structures. In a simplistic analysis we can accept this explanation because it follows the path from cause to effect. If we do an in-depth analysis of this process we can see that from the point of view of the analysis of the way in which this transformation is achieved quantum theory doesn't tell us anything about how this transformation takes place. Any process of change takes place in spatiotemporal coordinates. Transitions, defined as passing from one state to another, implicitly presume temporality (a duration for the process) that quantum theory does not explain. The atemporal characteristic of quantum theory reveals the limits for a profound temporal analysis. Jon Bell expresses himself firmly in favour of finding a solution in his book "Speakable and unspeakable in quantum mechanics", regarding the theoretical unification of the physical world: "The quantum phenomena do *not* exclude a uniform description of micro and macro worlds ... system and apparatus. It is *not* essential to introduce a vague division of the world of this kind" (Bell, 1987, p. 171).

He is on the side of those who believe in the theoretical unification by adopting a quantum vision of the macrocosm and considers de Broglie's concept of coexistence of waves and particles as a first step towards a unified theory an erroneous assumption.

Quantum theory has generated a theoretical schism and an antagonistic relationship between the microcosm and the macrocosm generating two worlds artificially separated without the introduction of criteria and limits based on theory. The thesis of theoretical unity of the physical world from the microcosm to the macrocosm in a unified theoretical system can still be achieved in the sense of reinterpretation of the microcosm and the description of the atomic and molecular structure in agreement with processual deterministic principles and the rigor of classical physics. This is the subsequent purpose of the theory that we want to implement.

5. Conclusions

Most epistemologies involved in debates on quantum theory avoids a position trenchant on dualism, determinism or discreet energy, realizing that such a position would conflict with the initial set of assumptions that are based and would require his conceptual reformulation. Their attitude is one of denial of quantum theory but a reinterpretation epistemological quantum concepts, they do not propose to change the paradigm of quantum but remains within it.

The epistemological debate on the theoretical foundations of quantum theory, should not be regarded as a problem outside of physics but as a major theme of his essence dichotomy between scientific knowledge and the epistemic is one such formal boundaries being mostly teaching theoretical knowledge the integration of the two parts must be understood in its complexity and entirety.

REFERENCES

Bell J., *Speakable and Unspeakable in Quantum Mechanics,* Cambridge University Press, pp. 170, 171 (1987).

Bunge M., *Matter and Mind a Philosophical Inquiry,* McGill University, Montreal, Canada, pp. 41, 56, 180 (2010).

Einstein A., American Journal of Physics, **33**, *5*, May, p.1 (1965).

Hodorogea M., *The End of Quantum Theory,* Axioma Print, Bucureşti, p.8 (2008).

Kuhn T.S., *The Structure of Scientific Revolutions,* 3rd Edition, Chicago: University of Chicago Press, pp. 10, 12, 49, 85 (1996).

Marmet P., *Absurdities in Modern Physics: a Solution, or, a Rational Interpretation of Modern Physics,* Éditions du Nordir, p.11 (1993).

Popper K.R., *Quantum Theory and the Schism in Physics from the Postscript to the Logic of Scientific Discovery,* Edited by W.W. Bartley, III, Hutchinson, p.173 (1982).

O NOUĂ PERSPECTIVĂ EPISTEMIOLOGICĂ PRIVIND PARADIGMA CUANTICĂ

(Rezumat)

Paradigma cuantică ca o metodă discretă de descrie a structurii lumii materiale, în mod fundamental bazată pe mecanica cuantică, este în contradicție cu teoria electromagnetică a luminii. Promovarea unei viziuni mecaniciste a fenomenelor subatomice a fost un pas înapoi în cunoaştere, prin revenirea la o teorie involută în ceea ce priveşte caracterul corpuscular al luminii, care a fost extins nepermis asupra structurii lumii materiale. Complexitatea procesualității lumii materiale ca o unitate între caracterul corpuscular discret şi manifestările electromagnetice continue dezvăluie o nouă teorie a realității fizice, într-o viziune complexă epistemică, în care cele două entități ontologice nu se exclud reciproc, ci mai degrabă ele coexistă într-un sistem teoretic unificat explicitat printr-o nouă abordare electrodinamică a lumii materiale care dezvăluie un univers spațiotemporal care este în acelaşi timp cognoscibil şi predictibil.

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 62 (66), Numărul 1, 2016
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

ON THE "PSEUDO INTELLIGENCE" OF THE SHAPE MEMORY ALLOYS THROUGH THEIR OWN "FRACTAL NEURAL NETWORK"

BY

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Received: March 3, 2016
Accepted for publication: April 19, 2016

Abstract. Assimilating the shape memory alloys as a complex systems, in the frame of Extendend Scale Relativity Theory, *i.e.* a fractal theory of motion with a arbitrary constant fractal dimension, some behaviours of such materials (shape memory effect, pseudo elasticity) are analyzed. In our opinion these behaviours can be associated to a pseudo intelligence of the shape memory alloys given by its "fractal neural network".

Keywords: shape memory alloys; fractal theory; pseudo elasticity.

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1. Introduction

Complex system is a very favorable medium for the appearance of instabilities (Chen, 1994; Morozov, 2012). These instabilities imply both chaos through different routes (intermittencies, quasi-periodicity, cascade of period-doubling bifurcations, sub-harmonic bifurcations, torus breakdown) and self-structuring through generation of complex structures (Dimitriu *et al*., 2007; Agop *et al.*, 2013; Dimitriu *et al*., 2013). According to the classical concepts, all theoretical complex system models (fluid models, kinetic models, etc.) assume that the dynamics of the complex system particles occur on continuous and differentiable curves, so that they can be described in terms of continuous and differentiable motion variables (energy, momentum, density, etc.). These motion variables are exclusively dependent on the spatial coordinates and time. In reality, the complex system dynamics proves to be much more complex and the above simplifications cannot be expected to explain all of the aspects of the complex system dynamics. However, this situation can still be standardized if we consider that the complexity of complex system interaction processes impose different time resolution scales, while the evolution complex system patterns lead to different degrees of freedom.

From the above mentioned arguments it results that the explication of the complex system dynamics can be based on the assumption that the motions of the complex system particles take place on continuous but non-differentiable curves (fractal curves) since, according to the procedures (Nottale, 1993; Nottale, 2011; Munceleanu *et al*., 2011; Agop & Magop, 2012), only a fractal curve is dependent on the scale resolution. Moreover, according to the methodology (Gouyet, 1992), through dynamics of special topologies which can implement evolution patterns in complex system, it can lead to various degrees of freedom. Such an assumption can be sustained by a typical example, related to the collision processes in complex system: between two successive collisions the trajectory of the complex system particle is a straight line that becomes non-differentiable at the impact point. Considering that all the collision impact points form an uncountable set of points, it results that the trajectories of the complex system particles become continuous but non-differentiable curves.

Since the non-differentiability (fractality) appears as a fundamental property of the complex system dynamics, it seems necessary to construct a corresponding non-differentiable complex system physics. We assume that the complexity of interactions in the complex system dynamics is replaced by non-differentiability (fractality). This topic (fractal motion) was systematically developed using either the Scale Relativity Theory (SRT) (Nottale, 1993; Nottale, 2011), or the Extended Scale Relativity Theory (ESRT), *i.e.* the Scale Relativity Theory in an arbitrary constant fractal dimension (Munceleanu *et al*., 2011; Agop & Magop, 2012).

Therefore, as described above, a Euclidian dynamics of a complex system with external constraints is replaced with the fractal dynamics of a complex system free of any external constraints. Practically, the motion with constraints in the Euclidian space, *i.e*. on continuous and differentiable curves, is replaced by a motion free of constraints in the fractal space, *i.e.* on continuous but non-differentiable curves. To do this, we have to admit the correspondence between the austenite-martensite phase transition in the shape memory alloys (Cisse *et al*., 2016; Stanciu, 2009) and the "fractality" of the motion trajectories of the complex system structural units (given by means of the scale resolution and the fractal dimension of the motion curve).

Let us reconsider the fractal hydrodynamic equations with an arbitrary constant fractal dimensions, *i.e.* the specific fractal momentum and fractal state density conservation laws (Munceleanu *et al*., 2011; Agop & Magop, 2012):

$$\partial_t \cdot v^l + v^i \cdot \partial_i \cdot v^l = -\partial^l \left(Q + u\right) \tag{1}$$

$$\partial_t \cdot \rho + \partial_l \cdot \left(\rho v^l\right) = 0 \tag{2}$$

with Q the specific fractal potential

$$Q = -2\lambda^2 \left(dt\right)^{\left(\frac{4}{D_F}\right)-2} \frac{\partial_i \cdot \partial^i \sqrt{\rho}}{\sqrt{\rho}} = \frac{u^l u_l}{2} - \lambda \left(dt\right)^{\left(\frac{2}{D_F}\right)-2} \partial_l \cdot u^l \tag{3}$$

In the previous relations

$$v^l = 2\lambda \left(dt\right)^{\left(\frac{2}{D_F}\right)-1} \partial^l \cdot s \tag{4}$$

is the standard classical speed which is differentiable and independent of the scale resolution *dt*,

$$u^l = \lambda \left(dt\right)^{\left(\frac{2}{D_F}\right)-1} \partial^l \ln\rho \tag{5}$$

is the non-standard speed which is non differentiable and scale resolution dependent, ρ the states density, U the external scalar potential, λ the fractal – non-fractal transition coefficient and D_F the fractal dimension of the motion curves. The speed fields v^l and u^l defines in the fractal space the complex speed field.

$$\tilde{V}^l = -2i\lambda \left(dt\right)^{\left(\frac{2}{D_F}\right)-1} \partial^l \ln\Psi = v^l - iu^l \tag{6}$$

with $\Psi = \sqrt{\rho}e^{is}$ the wave function, $\sqrt{\rho}$ the amplitude and s the phase. We remind that for time scales that prove to be larger if compared with the inverse of the highest Lyapunov exponent, the deterministic trajectories are replaces by a collection of potential paths and that the concept of "definite position" is substitute by that of the probability density. Thus Ψ does not have any physical significance but $|\Psi|^2 \equiv \rho$ has, as a probability density.

2. Fractal Hydrodynamic Equations

We assume that the motions of the complex system structural units take place on continuous but non-differentiable curves (fractal curves).

Any continuous but non-differentiable curve of the complex system particles (complex system fractal curve) is explicitly scale resolution dt dependent, *i.e.* , its length tends to infinity when dt tends to zero.

We mention that, mathematically speaking, a curve is non-differentiable if it satisfied the Lebesgue theorem, *i.e.* its length becomes infinite when the scale resolution goes to zero (Mandelbrot, 1982). Consequently, in this limit, a curve is as zig-zagged as one can imagine. Thus, it exhibits the property of self-similarity in every one of its points, which can be translated into a property of holography (every part reflects the whole) (Gouyet, 1992; Mandelbrot, 1982).

3. Solution of Fractal Hydrodynamic Equations for Fractal Static States

Let us now consider the fractal static state

$$\partial_t = 0, \quad v^l = 2\lambda(dt)^{\left(2/D_F\right)-1}\partial^l s = 0 \tag{7}$$

which implies "coherence states" (the structural units of the shape memory alloys (SMA) as a complex system) are in phase, *i.e.* s = *const*. Then the fractal hydrodynamics equations system (eqs. (1) - (3)) takes the following form:

$$\partial^l(Q+U) = 0, \quad -2\lambda^2(dt)^{\left(4/D_F\right)-2}\frac{\partial_i \cdot \partial^i\sqrt{\rho}}{\sqrt{\rho}} + U = E \tag{8}$$

where $E = \langle E\rangle$ represents the total specific fractal energy of the fractal static states. The states density conservation law, eq. (2), is identical satisfied.

In order to solve eq. (8) it should be known the functional dependence $U = U(\rho)$. The simplest choice is $U = C_1\rho$, with C_1 = const. In such condition eq. (2) in the one dimensional case and with the substitutions

$$\xi = \left(\frac{E}{2}\right)^{1/2}\frac{x}{\lambda(dt)^{\left(2/D_F\right)-1}}, \quad \rho^{1/2} = \left(\frac{E}{C_1}\right)^{1/2} u \qquad E > 0 \tag{9}$$

takes the shape of a Ginzburg – Landau equation type, which will be called Fractal Ginzburg Landau equation (FGLE)

$$\partial_{\xi\xi}u = u^3 - u \tag{10}$$

Generally this type of equations always admits the solution $u_c = \mathrm{th}\left[\left(\xi - \xi_0\right)/\sqrt{2}\right]$ (Cristescu, 2008). In the following we will build a new class of solution for the Ginzburg Landau fractal equation, through which u_c will be find as a particular solution. From such a perspective by multiplying both sides of the eq. (10) with $df / d\xi$ and performing the integration over ξ we obtain:

$$\partial_{\xi}u = \left(\frac{1}{2}u^4 - u^2 + C_2\right)^{1/2} \tag{11}$$

where

$$C_2 = \left[\left(\partial_{\xi}u\right)^2 - \frac{1}{2}u^4 + u^2\right]_{\xi=\xi_0} \tag{12}$$

Eq. (12) is obviously a restriction imposted on the order parameter *u*, showing the boundary conditions a further integration of eq. (11) leads to

$$\frac{\left(\xi - \xi_0\right)}{2^{1/2}} = \int_0^u \frac{du}{\left[\left(u_1^2 - u^2\right)\left(u_2^2 - u^2\right)\right]^{1/2}} \tag{13}$$

where

$$u_{1,2}^2 = 1 \mp \left(1 - 2C_2\right)^{1/2} \tag{14}$$

and ξ_0 a new constant of integration.

By the change of variable $w = u / u_1$ the integral (13) becomes:

$$\frac{u_2\left(\xi - \xi_0\right)}{2^{1/2}} = \int_0^w \frac{dw}{\left[\left(1 - w^2\right)\left(1 - k^2w^2\right)\right]^{1/2}} \tag{15}$$

where

$$k = \frac{u_1}{u_2} \tag{16}$$

Writing u_1 and u_2 in term of *k*:

$$u_1^2 = \frac{2k^2}{1+k^2} \ , \ u_2^2 = \frac{2k^2}{1+k^2} \tag{17}$$

the integral (15) becomes

$$\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} = \int_0^{w_1} \frac{dw}{\left[\left(1-w^2\right)\left(1-k^2w^2\right)\right]^{1/2}} \tag{18}$$

with the superior limit

$$w_1 = \left(\frac{1+k^2}{u}\right)^{1/2} \tag{19}$$

The integral (18) can be solved in terms of Jacobi elliptic function of argument $\left(1+k^2\right)^{-1/2}\left(\xi - \xi_0\right)$ and the modulus k (Bowman, 1955). Indeed, by the version of the integral (18) it results the Jacobi's elliptic function:

$$w = \mathrm{sn}\left[\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} : k\right] \tag{20}$$

of periods:

$$\omega_1 = 4\int_0^{\pi/2} \frac{d\varphi}{\left[\left(1-k^2\sin^2\varphi\right)\right]^{1/2}} \ , \ \omega_2 = 2i\int_0^{\pi/2} \frac{d\varphi}{\left[\left(1-k'^2\sin^2\varphi\right)\right]^{1/2}} \tag{21}$$

$$k^2 + k'^2 = 1$$

from which the solution is obtained by returning to the previous variable u :

$$u = \left(\frac{2k^2}{1+k^2}\right)^{1/2} \mathrm{sn}\left[\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} : k\right] \tag{22}$$

4. Conclusions

We can now discuss some conclusions:

i) The fractal state density takes the form:

$$u^2 = \left(\frac{2k^2}{1+k^2}\right)\mathrm{sn}^2\left[\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} : k\right] = \left(\frac{2k^2}{1+k^2}\right) - \left(\frac{2k^2}{1+k^2}\right)\mathrm{cn}^2\left[\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} : k\right] \tag{23}$$

while the specific fractal potential:

$$q = -u^{-1}\partial_{\xi\xi}u = \left(1 - u^2\right) \equiv \left(\frac{1 - k^2}{1 + k^2}\right) + \left(\frac{2k^2}{1 + k^2}\right)\mathrm{cn}^2\left[\frac{\xi - \xi_0}{\left(1 + k^2\right)^{1/2}} : k\right] \tag{24}$$

This mean that the functionality of a SMA is realized through cnoidals oscillatory modes of either fractal states density, or of the specific fractal potential;

ii) The explicit form of the oscillatory modes can be obtained through the degeneration of elliptic function cn in modulus k. Thus for

$$k \to 0\,,\;\; \omega_1 \to \frac{\pi}{2}\;,\;\; \omega_2 \to i\infty \tag{25}$$

we will have the degeneration

$$\begin{gathered} \mathrm{cn} \to \cos \\ n^2 \to 0 \\ q \to 1 \end{gathered} \tag{26}$$

For

$$k \to 1\,,\;\; \omega_1 \to \infty\,,\;\; \omega_2 \to i\pi \tag{27}$$

we will have the degeneration

$$\begin{gathered} \mathrm{cn} \to \mathrm{sech} \\ u^2 \to \mathrm{th}^2\left(\frac{\xi - \xi_0}{2^{1/2}}\right) \\ q \to \mathrm{sech}^2\left(\frac{\xi - \xi_0}{2^{1/2}}\right) \end{gathered} \tag{28}$$

The degeneration (25) with (26) corresponds to harmonic package, while the degeneration (27) with (28) to solitonic package. For $k \equiv 0$ harmonic sequence and for $k \equiv 1$ solitonic sequence are obtained. We note that the degeneration $k \equiv 1$ in solution (22) implies the standard solution u_c;

iii) In the classical theory the cnoidal oscillation modes are associated to a Toda lattice (Cristescu, 2008), *i.e.* a system of coupled nonlinear oscillators. Following the same procedure we will attribute the fractal cnoidal oscillation modes to the Toda type fractal lattice. Then the modulus k of the elliptic function cn becomes a measure of the coupling between the fractal oscillators of the Toda type fractal lattice: $k \to 0$ shows the total decoupling of the oscillators while $k \to 1$ shows the total coupling of the same oscillator. The functionality of the Toda type fractal lattice induces in the spectrum of the "fractal phononic field" an acoustic branch and an optical one. Such a dependence could explain why in the SMA materials are presented two phases, one called austenite (A), stable at high temperature and low stress and other one called martensite (M), metastable at low temperature and high stress;

iv) In the standard theory through the mapping of a Toda unidimensional lattice (Cristescu, 2008) it can be reduced to a specific neural network. Following the same procedure but applied to our case, one could find the fractal neural networks having specific functions.

REFERENCES

Agop M., Magop D., *New Considerations in Fractalspace-Time Theory,* in E.W. Mitchell and S.R. Murray (Editors), *Classification and Application of Fractals: New Research* (Nova Science, Hauppauge, NY), 131−159 (2012).

Agop M., Dimitriu D.G., Niculescu O., Poll E., Radu V., Phys. Scripta 87*,* 045501 (2013).

Bowman F., *Introduction to Elliptic Functions with Applications,* London, English University, Press (1955).

Chen F., *Introduction to Complex System Physics,* 2nd Edn. Springer, New York (1994).

Cisse C., Zaki W., Zimeb T.B., International Journal of Plasticity*, 76*, 244−284 (2016).

Cristescu C.P., *Dinamici neliniare şi haos*. *Fundamente teoretice şi aplicații*, Edit. Academiei Române, Bucureşti (2008).

Dimitriu D.G., Aflori M., Ivan L.M., Radu V., Poll E., Agop M., *Experimental and Theoretical Investigations of Plasma Multiple Double Layers and their Evolution to Chaos,* Plasma Sources Science and Technology, **22**, 035007 (2013).

Dimitriu G., Aflori M., Ivan L.M., Ionita C., Schrittwieser R.W., *Common Physical Mechanism for Concentric and Non-Concentric Multiple Double Layers in Plasma,* Plasma Physics and Controlled Fusion, **49**, *3*, 237−248 (2007).

Gouyet J.F., *Physique et Structures Fractales,* Masson, Paris (1992).

Mandelbrot B., *The Fractal Geometry of Nature,* Freeman, San Francisco (1982).

Morozov I., *Introduction to Complex System Dynamics,* CRC Press, Boca Raton, Florida (2012).

Munceleanu V., Păun V.P., Casian-Botez I., Agop M., Int. J. Bifurcation Chaos, **21**, 603 (2011).

Nottale L., *Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity,* World Scientific, Singapore (1993).

Nottale L., *Scale Relativity and Fractal Space-Time: A New Approach to Unifying Relativity and Quantum Mechanics,* Imperial College Press, London (2011).

Stanciu S., *Materiale cu memoria formei. Metode de investigații şi aplicații în tehnică,* Universitas, **XXI** (2009).

ASUPRA PSEUDOINTELIGENȚEI ALIAJELOR
CU MEMORIA FORMEI
DATĂ DE PROPRIA LOR REȚEA NEURONALĂ FRACTALĂ

(Rezumat)

Asimilând aliajele cu memoria formei ca sisteme complexe, în cadrul unei teorii fractale a mişcării în dimensiune fractală arbitrar constantă, sunt analizate câteva comportamente specifice ale acestor aliaje. În opinia noastră aceste comportamente sunt dictate de acea presupusă inteligență a materialelor cu memoria formei (pseudointeligență) asociată propriei lor rețele neuronale fractale.

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 62 (66), Numărul 1, 2016
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

A COMPUTER AIDED STUDY OF TWO PERPENDICULAR HARMONIC OSCILLATIONS OF THE SAME FREQUENCY

BY

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Received: March 3, 2016
Accepted for publication: April 15, 2016

Abstract. The goal of this paper is the study of two perpendicular harmonic oscillations of the same frequency using the HTML5 technology and the JavaScript language. With the aid of these technologies we developed the physics simulation for this experimental work. An experimental application is the determination of the speed of sound. This application is the subject of one of the laboratory classes of the Physics I course that is designated to the second semester of the first year at the Faculty of Civil Engineering and Building Services of "Gheorghe Asachi" Technical University of Iaşi. Using HTML5 and JavaScript language the students will be more motivated to work faster and a deeper understanding of the study of oscillatory motion will be developed. Moreover, the Matlab program is used for making the graphical representations.

Keywords: two perpendicular harmonic oscillations of the same frequency; Civil Engineering; HTML5; JavaScript; Matlab; simulations; physics phenomena.

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1. Introduction

Computational instruments and simulations are more and more widely used and many teachers deal with them to improve the teaching-learning process (http://phet.colorado.edu; http://wildcat.phys.northwestern.edu; www.myphysicslab.com; http://virlab.virginia.edu; http://www.java.com/en; Covatariu & Covatariu, 2008).

Traditionally, working in the physics laboratory means to work in a controlled environment where the information can be introduced sequentially by using the laboratory equipment. The target for each lab is to pick up some experimental data and compute the value of some physical required quantities together with making graphical representations where necessary. An alternative to experimental works are the physics simulations. Along the years, our experience in using computational instruments for improving our Physics courses and laboratories pointed out that traditional methods in teaching-learning Physics unified with computer simulations can be an important tool that allows students to explore modern Physics and better understand the physical phenomena and engineering studies (Smith & Pollard, 1986; Andaloro *et al.*, 1991; Wisman & Forinash, 2008; Gould *et al.*, 2007).

In this way, the students will improve their learning process and obtain good marks at exams. In the last years we used to work with programs like Adobe Flash (adobe.com/products/flash), Maple, Mathematica and Matlab (www.maplesoft.com; www.wolfram.com; www.mathworks.com). Also, in the future, we want to extend our course to include more physics phenomena and experiments based on simulations packages.

Our Physics 1 course runs one semester and is a 3 credits course that consists in lecture combined with advanced laboratories that provide the description of the most important physics methods with applications in Civil Engineering. During the laboratories, students study the two perpendicular harmonic oscillations of the same frequency, damped oscillations, standing waves, electromagnetic waves and some optical phenomena such as light interference and diffraction.

In this paper, as an example of physics simulation the authors use HTML5 and the JavaScript language (https://developer.mozilla.org/en-US/docs/Web/Guide/HTML/HTML5; https://www.javascript.com) for the study of two perpendicular harmonic oscillations of the same frequency. An application of this physics laboratory is the evaluation of the speed of sound. Also, for plotting the experimental data the Matlab program can be used.

2. Two Perpendicular Harmonic Oscillations of the Same Frequency

One of the two hours physics laboratories of the second semester at the Faculty of Civil Engineering and Building Services of “Gheorghe Asachi” Technical University is focused on the study of two perpendicular harmonic oscillations of the same frequency (Neagu *et al*., 2001). Together with the experimental method we also use the physics simulation that is elaborated using the HTML5 technology and the JavaScript language. The experiment conveys how the speed of sound can be calculated using two perpendicular harmonic oscillations of the same frequency.

In the next we present the study of two perpendicular harmonic oscillations of the same frequency. Let us consider that a material point simultaneously undergoes two harmonic oscillations, one along *Ox*, the other along *Oy:*

$$x = A\sin\omega t\,, \tag{1}$$

$$y = B\sin(\omega t + \varphi)\,. \tag{2}$$

In order to obtain the trajectory equation, we leave out the time from the relations (1) and (2).

As a result, relation (1) becomes:

$$\sin\omega t = \frac{x}{A} \Rightarrow \cos\omega t = \sqrt{1-\sin^2\omega t} = \sqrt{1-\frac{x^2}{A^2}}\,. \tag{3}$$

Combining (1) and (2) one obtains:

$$y = B\sin\omega t\cos\varphi + B\cos\omega t\sin\varphi\,,$$

$$y = B\frac{x}{A}\cos\varphi + B\sqrt{1-\frac{x^2}{A^2}}\sin\varphi\,,$$

$$y - B\frac{x}{A}\cos\varphi = B\sqrt{1-\frac{x^2}{A^2}}\sin\varphi\,. \tag{4}$$

By squaring this relation, one gets:

$$\frac{y^2}{B^2} + \frac{x^2}{A^2} - \frac{2xy}{AB}\cos\varphi = \sin^2\varphi\,. \tag{5}$$

The trajectory is an ellipse. The consecutive motion is an elliptical periodical motion (Fig. 1).

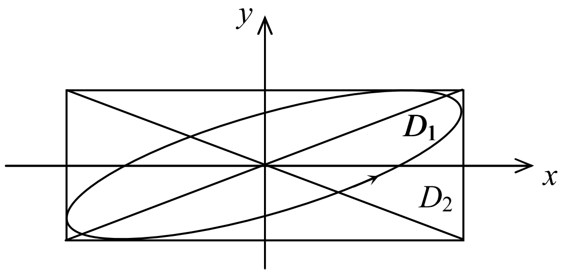

Fig. 1 – The elliptical periodical motion.

Particular Cases

By denoting with $\varphi = \varphi_2 - \varphi_1$ the phase difference between the oscillations we have the following particular cases.

Case 1) If $\varphi_2 - \varphi_1 = 2k\pi$, from relation (5) it follows:

$$\frac{y^2}{B^2} + \frac{x^2}{A^2} - \frac{2xy}{AB} = 0 \Rightarrow \frac{x}{A} = \frac{y}{B} . \tag{6}$$

The consecutive motion is a harmonic oscillatory motion along the line D_1 of slope $\frac{B}{A}$ that represents the first diagonal from the amplitude rectangular. This is the case of linear polarization.

Case 2) If $\varphi_2 - \varphi_1 = (2k+1)\pi$, from relation (5) one gets:

$$\frac{x}{A} = -\frac{y}{B} . \tag{7}$$

The consecutive motion is a harmonic oscillatory motion along the line D_2 of slope $-\frac{B}{A}$. Also, one gets linear polarization.

Case 3) If $\varphi_2 - \varphi_1 = (2k+1)\pi/2$, from relation (5) we have:

$$\frac{y^2}{B^2} + \frac{x^2}{A^2} = 1 . \tag{8}$$

The motion is periodical on an ellipse around an axis. For $\varphi_2 - \varphi_1 = \frac{\pi}{2}$ the motion is clockwise and we have elliptical polarization (right-handed polarization) and for $\varphi_2 - \varphi_1 = 3\pi/2$ the motion is counter-clockwise, with elliptical polarization (left-handed polarization).

A particular case is $A = B$ and the ellipse becomes a circle:

$$x^2 + y^2 = A^2 . \quad (9)$$

In this case we have circular polarization.

Reciprocally, any circular periodical motion can be decomposed in two cross harmonic oscillations, of similar ν and A, and with the phase difference $(2k+1)\pi/2$.

The laboratory equipment consists of electrical connections, signal (frequency) generator, oscilloscope, microphone, speakerphone, a mobile tube and a fixed tube. The tubes are filled with air.

An application of this physics lab is the evaluation of the speed of sound. By keeping a fixed value of the frequency ν at the generator the students will vary the distance by moving the mobile tube and will mark the positions where they obtain a difference of phase equals to π and 2π, respectively (in these cases they collect experimental data between the lines D_1 and D_2, and between the lines D_1, D_2 and again D_1, respectively). In this way the students are able to check experimentally the relationship between the speed of sound and the variation of the length of the air column and frequency ν at the generator.

They calculate the speed of sound with the relation

$$v = 2(l_2 - l_1)\nu , \quad (10)$$

for a difference of phase equal to π. For the difference of phase equal to 2π they use the relation

$$v = (l_2 - l_1)\nu . \quad (11)$$

An important task is to take and record measurements for at least ten different frequencies.

If the case of different frequencies the consecutive trajectory has a more complicated form. If the frequency ratio is a rational number, $\nu_1 / \nu_2 = n_1 / n_2$, $n_1, n_2 \in N$ what we obtain are closed trajectories, called Lissajous figures.

In the physics laboratory the experimental data are collected using the laboratory equipment that has been previously presented. In the case of the physics simulation the Fig. 3 includes a similar configuration detailed bellow.

3. HTML5 and JavaScript Simulation for Two Perpendicular Harmonic Oscillations of the Same Frequency. Plotting the Experimental Data

Working in a traditionally physics lab the students need to complete all the activities for reaching the goal of the lab. This consists in picking up by hand the experimental data, make calculations and graphs. A step along the line of improving the activities in the physics laboratories is the use of physics simulations. A physics simulation makes experimenting and plotting faster and fosters commitment and a feeling of involvement to the advantage of learning. Moreover, a physics simulation has a positive impact on learning process and also an important educational purpose, to raise curiosity for the used computational resources and for the physical phenomena under study.

We have implemented the HTML5 technology and the JavaScript language in the Physics laboratories by developing the simulation for the study of two perpendicular harmonic oscillations of the same frequency. Moreover, the experiment allows students to calculate the speed of sound.

The graphical representations are obtained using the Matlab program. Engineers and scientists worldwide use the Matlab program because its many capabilities like numeric computation, data analysis and visualization, programming and algorithm development, graphics, control system design and analysis.

In the following we present the physics simulation for the study of two perpendicular harmonic oscillations of the same frequency and the commands used for plotting the experimental data. For plotting data the Matlab program is used and as an example the graphs of the consecutive motion are showed in the Fig. 2. We also show a sequence of code in the Matlab program which has been used for plotting the trajectories of the oscillatory motion, as follows.

```
% waves plot for
% x=A*sin(omega*t)
% y=B*sin(omega*t+phi)
A = 4;
omega = 2;
B = 6;
phi1 = 0;
t= 1:0.05:10
x = A*sin(omega*t)
y = B*sin(omega*t+phi1*pi)
figure()
plot(x,y)
axis([-4 4 -6 6])
title([ ' {\fontname{symbol}j}',' = ',char(num2str(b3a))])
drawaxis(gca, 'x', 0, 'movelabel', 1,  'y', 0, 'movelabel', 1)
```

We notice that the students also use the Matlab program for plotting the picked up experimental data.

The most frequently solutions used by an animation into a browser are flash animations based on Adobe and HTML5 technology that are programming languages adopted by the World Wide Web Consortium (W3C). Each method has advantages and disadvantages.

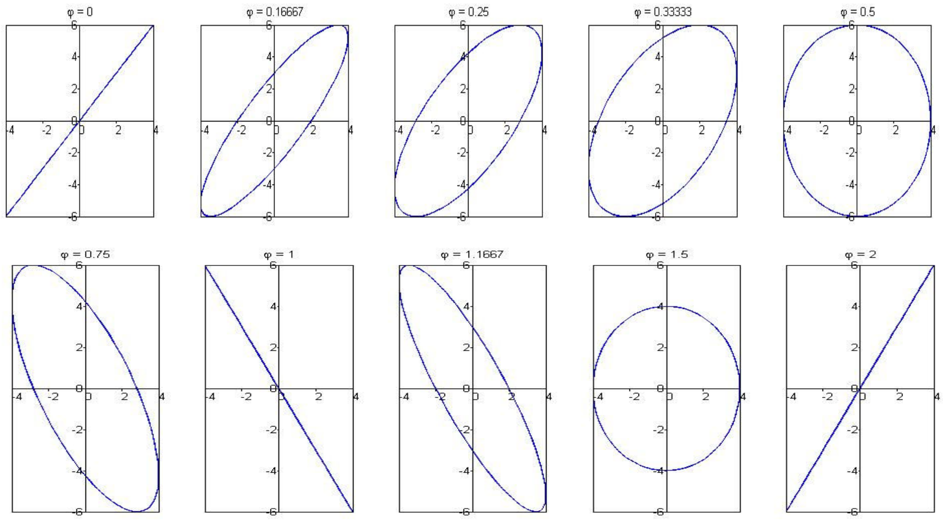

Fig. 2 – Graphs of the consecutive motion by using the Matlab code.

The advantage of a plugin such as Adobe Flash Player is that the animation is not dependently on the browser version. The disadvantage is that it is an exclusive technology and the company that owns the software can decide how it wants in its evolution. This is an inconvenient because the applications cannot run on certain operating systems or mobile devices.

HTML5 is a technology that uses open standards agreed by the international organizations. Additional plugins are not required for any application that runs in the browser natively. However, this technology has some difficulties because the implementation of various works in different browsers is not standardized and developers must solve various compatibility issues.

To achieve virtual animation of this experiment HTML5 was chosen.

A static image of equipment's was reproduced in the Fig. 3 by the virtual application that consists of electrical connections, signal (frequency) generator, oscilloscope, microphone, speakerphone, a mobile tube and a fixed tube. The static image presents the major advantage of being easy to use into a web application.

There are 3 HTML canvas elements of the static image whose charts were realized. Two are used for measuring instruments and the last one for sound animation on the microphone – speakerphone layout. Besides all of these elements, different control tools are used as: on-off switches, amplitude and frequency controllers and simulated microphone that can be horizontally moved using the mouse.

Moreover, external JavaScript libraries were used to achieve this virtual application as follows:

− Jquery.js and jquery-ui.js user interface were used to access the webpage and may succeed the functionality of the microphone movements;

− Alertify.js and tooltipster.js have an aesthetic role. Also, were used to show the error messages in case of browser incompatibility (alertify.js) or to display a help message balloon (tooltipster.js);

− Modernizr.js is used to detect the browser compatibility of the core technology;

− Require.js is used to organize the JavaScript code.

All listed libraries are free for use and adaptation. Furthermore, specific code sequences were realized in different files. Below, the functionality is briefly listed:

− Graph.js implements specific methods to achieve the graphical representation. Basic functions are used in HTML5 to draw a canvas element (MmveTo, lineTo etc.). To obtain the appearance of Lissajous figures the *xliss(t)* and *yliss(t)* mathematical functions were used. The results obtained by *xliss(t)* and *yliss(t)*, respectively are transmitted to *drawParametricCurve* function that is defined in the *graph.js* module beside with other required plotting procedures.

```
Graph.prototype.drawParametricCurve = function(xeq, yeq, color, thickness) {
        var context = this.context;
        context.save();
        this.clear();
        this.transformContext();
        context.beginPath();
        for(var t = 0; t <= 3; t += 0.003) {
          context.lineTo(xeq(t), yeq(t));
        }
        context.restore();
        context.lineJoin = 'round';
        context.lineWidth = thickness;
        context.strokeStyle = color;
        context.stroke();
        context.restore();
  };
```

− Amplitude.js performs the amplitude modification by position adjustment of knob on the gauge;

− Frequency.js has the same functionality as Amplitude.js, applied for frequency adjustments;

− Lissajou.js and Signal.js use functions defined in Graph.js and implement the mathematical structure for the signal generator and oscilloscope. These functions accept three parameters: the amplitude, the frequency and the phase;

```
define(['graph','jquery'], function (Graph,$) {
        function Lissajou(displayId){
                this.myGraph = new Graph({
                  canvasId: displayId,
                  minX: -10,
                  minY: -10,
                  maxX: 10,
                  maxY: 10,
                  unitsPerTick: 1
                });
        }
        Lissajou.prototype.drawLissajou = function(amp, frecv, phase){
           function xliss(t) {
             return amp*Math.sin(2*Math.PI*frecv*t);
           }
           function yliss(t) {
             return amp*Math.sin(2*Math.PI*frecv*t+phase);
           }
          this.myGraph.drawParametricCurve(xliss,yliss,'#ffff66',4);
        };
        Lissajou.prototype.drawSpot = function(){
                this.myGraph.drawSpot('#ffff66');
        };
        Lissajou.prototype.clear = function(){
                this.myGraph.clear();
        };
        return Lissajou;
        });
```

− Phase.js uses the microphone position to calculate the phase difference between the oscilloscope signals;

− Colorgen.js and Wave.js are used to achieve the animation of the sound into the layout of speakerphone – microphone.

Thus, the end user can start the generator and the oscilloscope (Fig. 3). The amplitude and frequency can be adjusted by using the sliders from the signal generator. The phase shift changes by varying the position of the microphone and appearance of Lissajous figure occurs. All virtual changes and adjustments are carried out to observe a real experiment in a similar way.

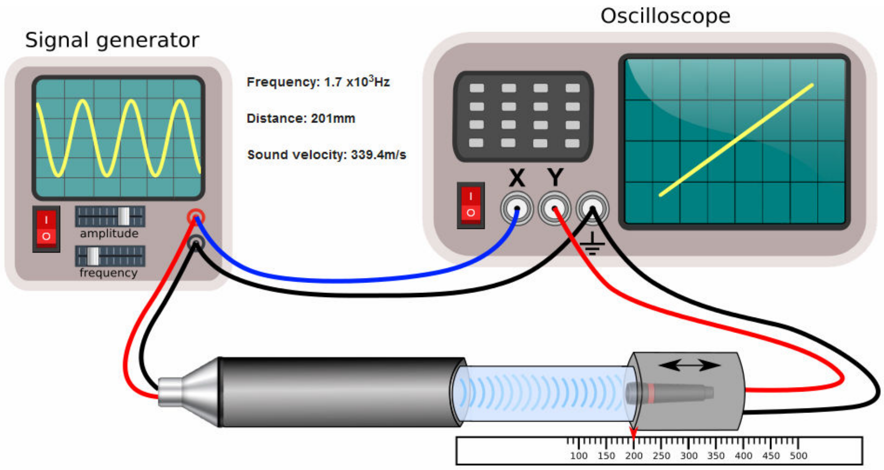

Fig. 3 – Experimental setup animation.

4. Conclusions

Physics simulations are interesting and powerful tools in education because support tasks like easy learning and study of physical phenomena. They do not impose much burden as the traditional activity in the physics laboratory and in this way, the time spent for study physical phenomena and laws can be reduced. In addition, working with them does not require a deeper understanding of the simulation program. Also, physics simulations provide the students with important steps in their educational progress.

In recent years, in order to support and facilitate the learning of physics phenomena we have developed some physics simulations (Radinschi *et al*., 2008; Radinschi & Damoc, 2008; Radinschi & Aignatoaie, 2010). The newest is the simulation used for the study of two perpendicular harmonic oscillations of the same frequency and for the evaluation of the speed of sound. Using this physics simulation the students improved their skills of working in the physics laboratory and also have obtained better performance in the learning process. This is because they have only to follow the defined instructions for using the physics simulation and make a comparison with the picked up experimental data. Likewise, in this way they learn by observing the simulation of the physics phenomena and, also working in the physics laboratory. Moreover, the physics simulation is posted on the website of our faculty at the address http://server.ce.tuiasi.ro/~radinschi/simulation/default.html in the virtual physics laboratory replacing the face to face learning and there will not be time and distance limitation. We consider that in this way the self-study and Internet based discussions are powerful tools for improving the students' learning enthusiasm. As for the future work, for providing the students with an adequate environment for working in the physics laboratory we want to address other computational programs and elaborate more physics simulations.

Acknowledgements. The authors are thankful to the anonymous referee for his helpful comments and suggestions.

REFERENCES

Andaloro G., Donzelli V., Sperandeo-Mineo R.M., *Modelling in Physics Teaching: the Role of Computer Simulation,* International Journal of Science Education, **13**, *3*, 243−254, 1991.

Covatariu G., Covatariu D., *Development of Information Technology in Civil Engineering,* Acta Technica Napocensis: Civil Engineering & Architecture, **51**, *1*, 105−114, Proceedings of the International Conference Constructions, May 9-10, Cluj-Napoca, 2008.

Gould H., Tobochnik J., Christian W., *An Introduction to Computer Simulations Methods: Applications to Physical Systems,* 3rd. Ed., Addison-Wesley, 2007.

Neagu M.R., Picos S., Carpinschi N., Mirea L., Radinschi I., *Guideline for Physics Laboratories,* "Gh. Asachi", Publishing House, Iaşi, 2001.

Radinschi I., Damoc C., Cehan A., Cehan V., *Computer Simulations of Physics Phenomena Using Flash,* Proc. of the 5th International Conference on Hands-on Science Formal and Informal Science Education, HSCI 2008, Espaço Ciência, Olinda-Recife, Brasil, 147−152, 2008.

Radinschi I., Damoc C., *Computer Simulations for Physics Laboratory,* Proc. of the Sixth International Symposium "Computational Civil Engineering 2008", CCE 2008, 441−447, 2008.

Radinschi I., Aignatoaie, B., *Efficiency of Using a Virtual Physics Laboratory,* Bul. Inst. Polit. Iași, **LVI (LX)**, *4*, s. Mathematics. Theoretical Mechanics. Physics, 141−146 (2010).

Smith P.R., Pollard D., *The Role of Computer Simulations in Engineering Education,* Computers & Education, **10**, *3*, 335−340, July 1986.

Wisman R.F., Forinash K., *Science in Your Pocket,* Proc. of the 5th International Conference on Hands-on Science Formal and Informal Science Education, HSCI 2008, Espaço Ciência, Olinda-Recife, Brasil, 180−187, 2008.

www.adobe.com/products/flash

www.developer.mozilla.org/en-US/docs/Web/Guide/HTML/HTML5

www.java.com/en

www.javascript.com/

www.maplesoft.com/

www.mathworks.com/products/matlab

www.myphysicslab.com, http://virlab.virginia.edu

www.phet.colorado.edu, http://wildcat.phys.northwestern.edu

www.wolfram.com

STUDIU ASISTAT DE CALCULATOR PENTRU SIMULAREA A DOUĂ OSCILAŢII PERPENDICULARE DE ACEEAŞI FRECVENŢĂ

(Rezumat)

Scopul acestei lucrări este studiul a două oscilaţii armonice perpendiculare de aceeaşi frecvenţă folosind tehnologiile de programare HTML5 şi JavaScript. Cu ajutorul acestor tehnologii s-a dezvoltat o simulare a proceselor fizice ce au loc în lucrarea experimentală. O aplicaţie experimentală este determinarea vitezei sunetului în aer. Această aplicaţie reprezintă subiectul unei lucrări de laborator aferente cursului de Fizică din semestrul doi al primului an de studiu al Facultăţii de Construcţii şi Instalaţii a Universităţii Tehnice "Gheorghe Asachi" din Iaşi. Utilizând tehnologiile HTML5 şi JavaScript studenţii vor fi mai motivaţi să lucreze mai rapid şi vor beneficia de o înţelegere optimă a fenomenului fizic privind mişcarea oscilatorie. În plus, pentru reprezentările grafice a fost utilizat programul Matlab.

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 62 (66), Numărul 1, 2016
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

DIFFERENTIABLE AND NON-DIFFERENTIABLE CELLULAR NEURAL NETWORKS WITH IMPLICATIONS IN THE BACTERIAL GROWTH PROCESS. A MATHEMATICAL MODEL (I)

BY

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Received: January 11, 2016
Accepted for publication: April 18, 2016

Abstract. In the framework of the Extended Scale Relativity Theory, the Schrödinger type geodesics and non-differentiable hydrodynamic model are given. In the one-dimensional case of the non-differentiable hydrodynamic model, an analytical solution is obtained in the form of cnoidal oscillation modes. Associating a Toda lattice to the cnoidal oscillation modes, two types of cellular neural networks result by mapping: a differentiable cellular neural network and a non-differentiable cellular neural network. This model can be applied to bacterial growth process.

Keywords: differentiable cellular neural network; non-differentiable cellular neural network; coherence; Toda lattice; cnoidal oscillation modes; soliton; wave; neuron.

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1. Introduction

Many phenomena with complex patterns and structures are widely observed in the brain. These phenomena are some manifestations of a multidisciplinary paradigm called emergence or complexity. They share a common unifying principle of dynamic arrays, namely, interconnections of a sufficiently large number of simple dynamic units can exibit extremely complex and self-organizing behaviors (Karayiannis & Venetsanopoulos, 1993; Chow, 2007).

Standard theoretical models of complex systems dynamics are sophisticated and ambiguous (Flake, 1998; Mitchell, 2009). However, such situation can be simplified if we consider that complexities in interaction processes impose various time resolution scales and the evolution pattern leads to different freedom degrees. To develop new theoretical models, we have to admit that the complex system entities with chaotic behaviours can achieve self-similarity (space-time structures can appear) associated with strong fluctuations at all possible space-time scales. Then, for time scales that prove to be larger if compared with the inverse of the highest Lyapunov exponent, the deterministic trajectories are replaced by a collection of potential routes. In its turn, the concept of "definite positions" is peplaced by that of probability density.

Thus, non-differentiability appears as a universal property of complex systems and, moreover, it is necessary to create non-diffentiability physics of complex systems. Under such circumstances, if we consider that the complexity of interactions in the dynamics of complex systems is replaced by non-differentiability, it is no longer necessary to use the whole classical arsenal of quantities from standard physics (differentiable physics). This topic was developed using either the Scale Relativity Theory (SRT) (Nottale, 2011) and Extended Scale Relativity Theory (ESRT) *i.e.*, SRT with arbitrary constant fractal dimension. On the fundaments of such model and some of its implications, the reader can refer to Gottlieb *et al.*, 2006; Gurlui *et al.*, 2006, Timofte *et al.*, 2011. According to these models, the dynamics of complex system quasi-particles takes place on continuous but non-differentiable curves (fractal curves) so that all physical phenomena involved depend not only on space-time coordinates but also on space-time scale resolution. That is why physical quantities describing the dynamics of complex systems can be considered as fractal functions (Nottale, 2011; Mandelbrot, 1983). Moreover, according to geodesics in a non-differentiable (fractal) space, the complex system quasi-particles may be reduced to, and identified with, their own trajectories (*i.e.*, their geodesics) so that the complex system should behave as a special "fluid" lacking interactions - fractal fluid (Nottale, 2011; Timofte *et al.*, 2011).

In the present paper, we consider that the nervous influxe (through brain's neuronal network) takes place on continuous but non-differentiable curves in the hydrodynamic variant of scale relativity (with arbitrary constant fractal dimension). Dynamics types that are compatible with such frame of a cellular neural network are analyzed. In a such context some implication of the mathematical model in the bacterial growth process are achived.

2. Results and Discussions

2.1. Brief Remainder of the Theory

We can simplify the dynamics in complex systems assuming that its entities (structural units) (Mitchell, 2009) move on continuous but non-differentiable curves. Once accepted such a hypothesis, the dynamics of the entities is given by the non-differentiable operator $\hat{d}/dt$ (Timofte *et al.*, 2011).

$$\frac{\hat{d}}{dt}=\frac{\partial}{\partial t}+\hat{\boldsymbol{V}}\cdot\nabla-iD\left(dt\right)^{(2/D_F)-1}\Delta \tag{1}$$

where

$$\hat{\boldsymbol{V}}=\boldsymbol{V}_D-i\boldsymbol{V}_F \tag{2}$$

is the complex velocity, $\mathbf{V}_D$ is the differentiable and scale resolution independent velocity, $\mathbf{V}_F$ is the non-differentiable and resolution scale dependent velocity, $\hat{\boldsymbol{V}}\cdot\nabla$ is the convective term, $D(dt)^{(2/D_F)-1}\Delta$ is the dissipative term, ∇ is the nabla operator, Δ is the Laplace operator, D_F is the fractal dimension of the motion curve, dt is the scale resolution and D is a specific coefficient associated to the differentiable-non-differentiable transition. For D_F any definition can be used (the Hausdorff-Besikovici fractal dimension, the Kolmogorov fractal dimension etc. (Mandelbrot, 1983)), but once accepted such a definition for D_F, it has to be maintained over the entire analysis of the complex system dynamics. Applying the non-differentiable operator (1) to the complex speed (2) and accepting the scale covariance principle in the form:

$$\frac{\hat{d}\hat{\boldsymbol{V}}}{dt}=-\nabla \mathrm{U} \tag{3}$$

we obtain the motion equation

$$\frac{\hat{d}\hat{\boldsymbol{V}}}{dt}=\frac{\partial\hat{\boldsymbol{V}}}{\partial t}+\left(\hat{\boldsymbol{V}}\cdot\nabla\right)\hat{\boldsymbol{V}}-iD\left(dt\right)^{(2/D_F)-1}\Delta\hat{\boldsymbol{V}}=-\nabla U \tag{4}$$

where U is the external scalar potential. It means that at any point of a non-differentiable path, the local acceleration term, $\partial_t\hat{\boldsymbol{V}}$, the non-linearly (convective) term, $(\hat{V}\cdot\nabla)\hat{V}$ the dissipative term, $D(dt)^{(2/D_F)-1}\Delta\hat{V}$ and external force, ∇U makes their balance. Therefore, the complex structure dynamics can be assimilated with the dynamics of a "rheological" fluid. This dynamics is described by the complex velocities field, $\hat{V}$, the complex acceleration field, $\hat{d}\hat{V}/dt$ etc. and by the imaginary viscosity type coefficient, $iD(dt)^{(2/D_F)-1}$.

For irrotational motions of the complex system entities:

$$\nabla\times\hat{V}=0,\nabla\times V_D=0,\nabla\times V_F=0 \tag{5}$$

we can choose $\hat{V}$ of the form

$$\hat{V}=-2iD\left(dt\right)^{(2/D_F)-1}\nabla\ln\Psi \tag{6}$$

where $\varphi\equiv\ln\Psi$ is the velocity scalar potential. By substituting (6) in (4) and using the method described in (Timofte *et al*., 2011), it results:

$$\frac{d\hat{V}}{dt}=-2iD\left(dt\right)^{(2/D_F)-1}\nabla\left[\frac{\partial\ln\Psi}{\partial t}-2iD\left(dt\right)^{(2/D_F)-1}\frac{\nabla\Psi}{\Psi}\right]=-\nabla\mathrm{U} \tag{7}$$

This equation can be integrated and yields:

$$D^2\left(dt\right)^{(4/D_F)-2}\Delta\Psi+iD\left(dt\right)^{(2/D_F)-1}\frac{\partial\Psi}{\partial t}-\frac{\mathrm{U}}{2}\Psi=0 \tag{8}$$

up to an arbitrary phase vector which may be set to zero by a suitable choice of the phase of Ψ. For motions of complex system entities on Peano's curves, $D_F=2$, the eq. (8) takes the Nottale's form:

$$D_N^2\Delta\Psi+iD_N\frac{\partial\Psi}{\partial t}-\frac{\mathrm{U}}{2}\Psi=0,\ D\equiv D_N \tag{9}$$

Moreover, for motions of complex systems on Peano curves, $D_F=2$, at Compton scale, $D=\hbar/2m_0$, with $\hbar$, the reduced Planck constant and m_0 the rest mass of the complex system entities, the relation (8) becomes the standard Schrödinger equation (Phillips, 2003):

$$\frac{\hbar^2}{2m_0}\Delta\Psi+i\hbar\frac{\partial\Psi}{\partial t}-U\Psi=0 \tag{10}$$

If $\Psi = \sqrt{\rho} e^{iS}$, with $\sqrt{\rho}$ the amplitude and S the phase of Ψ, the complex velocity field (6) takes the form

$$\hat{\boldsymbol{V}} = 2D\left(dt\right)^{(2/D_F)-1} \nabla \mathrm{S} - iD\left(dt\right)^{(2/D_F)-1} \nabla \ln\rho \tag{11}$$

$$\boldsymbol{V}_D = 2D\left(dt\right)^{(2/D_F)-1} \nabla \mathrm{S} \tag{12}$$

$$\boldsymbol{V}_F = D\left(dt\right)^{(2/D_F)-1} \nabla \ln\rho \tag{13}$$

By substituting (11)‒(13) in (4) and separating the real and the imaginary parts, up to an arbitrary phase factor which may be set at zero by a suitable choice of the phase of Ψ, we obtain:

$$\frac{\partial \boldsymbol{V}_D}{\partial t} + \left(\boldsymbol{V}_D \cdot \nabla\right)\boldsymbol{V}_D = -\nabla\left(Q + U\right) \tag{14}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \boldsymbol{V}_D\right) = 0 \tag{15}$$

with Q the specific non-differentiable potential

$$Q = -2D^2\left(dt\right)^{(4/D_F)-2} \frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} = -\frac{\boldsymbol{V}_F^2}{2} - D\left(dt\right)^{(2/D_F)-1} \nabla \cdot \boldsymbol{V}_F \tag{16}$$

The eq. (14) represents the specific momentum conservation law, while eq. (15) represents the states density conservation law. The eqs. (14)‒(16) define the non-differentiable hydrodynamics model (NDHM).

The following conclusions are obvious:

i) Any entity of the complex system is in a permanent interaction with the non-differentiable medium through the specific non-differentiable potential (16);

ii) The non-differentiable medium is identified with a non-differentiable fluid described by the specific momentum and states density conservation laws;

iii) The non-differentiable speed $\mathbf{V}_F$ does not represent actual mechanical motion, but it contributes to the transfer of specific momentum and the concentration of energy. This may be seen clearly from the absence of $\mathbf{V}_F$ from the states density conservation law (15) and from its role in the variational principle (Nottale, 2011);

iv) Any interpretation of Q should take cognizance of the "self" nature of the specific momentum transfer. While the energy is stored in the form of the mass motion and potential energy (as clasically is), some is available elsewhere and only the total is conserved. It is the conservation of the energy and specific momentum that ensure reversibility and the existence of eigenstates, but denies a Brownian motion of interaction with an external medium (Nottale, 2011);

v) Since the position vector of the complex system entity is assimilated to a stochastic process of Wiener type (Mandelbrot, 1983), Ψ is not only the scalar potential of complex speed (through $\ln\Psi$) in the frame of non-differentiable hydrodynamics, but also density of probability (through $|\Psi|^2$) in the frame of a Schrödinger type theory. It results the equivalence between the formalism of the non-differentiable hydrodynamics and the ones of the Schrödinger type equation. Moreover, the chaoticity, either through turbulence in the non-differentiable hydrodynamics approach or through stochasticization in the Schrödinger type approach, is generated by the non-differentiability of the movement trajectories in a fractal space.

2.2. Cnoidal Modes, Toda Lattices and Cellular Neural Networks

In one dimensional case, the eqs. (14)−(16) in non-dimensional coordinates

$$\omega t = \tau, kx = \xi, \theta = \xi - M\tau$$

$$\frac{\boldsymbol{V}_D}{\boldsymbol{V}_{0D}} = v, \frac{\rho}{\rho_0} = N, \frac{k\boldsymbol{V}_{0D}}{\omega} = 1 \tag{17}$$

for $U = -E\rho$ with $E =$ const., through integration and by eliminating the velocities field, become

$$\frac{k^2 D^2 (dt)^{(4/D_F)-2}}{\boldsymbol{V}_{0D}^2} \cdot \frac{1}{N^{\frac{1}{2}}} \cdot \frac{d^2 N^{\frac{1}{2}}}{d\theta^2} = \frac{E\rho_0}{\boldsymbol{V}_{0D}} N + \frac{M^2}{2} - \frac{c_1^2}{2N^2} + c_2 \tag{18}$$

In the above relations ω is a critical pusation, k is the inverse of a critical length, V_{0D} is a critical velocity, ρ_0 is a critical density, M is an equivalent of Mach number and c_1, c_2 are two integration constants.

Using the standard procedure (Timofte *et al.*, 2011), one obtains the solution

$$N = \overline{N} + 2a\left[\frac{E(s)}{K(s)} - 1\right] + 2a \cdot cn^2\left[\frac{\sqrt{a}}{s}(\theta - \theta_0); s\right] \tag{19}$$

In relations (19), $\overline{N}$ is the average value of the normalized states density, a is the amplitude, s is the modulus of the complete elliptical integrals of the first and second kind, $K(s)$ and respectively $E(s)$, and cn is the Jacobi elliptical function of modulus s (Armitage, 2006) and argument $\sqrt{a}(\theta-\theta_0)/s$. The modulus s is a control parameter of non-linearity, depending on both the intrinsic properties of the complex system and of the dynamics regimes. Therefore, the dynamics of the complex system is achieved through space-time cnoidal oscillation modes of the normalized states density. The cnoidal oscillation modes are associated to the Toda lattice (Toda, 1981).

Let us note that the elliptic function cn is double periodical, the real period being dependent of the modulus s, while the imaginary one is dependent of the complementary modulus s' with conditioning $s^2+s'^2\equiv 1$. Then, by (19), two solutions are actually explained, one of them being specific to the states density at differentiable scale induced by the modulus s and the other one being specific to the states density at non-differentiable scale induced by the complementary modulus s'. To these two solutions are now associated two Toda type sub-networks Toda, one of them being specific to the differentiable scale (which will be called *differentiable Toda lattice* (DTL)) and the other one being specific to the non-differentiable scale (which will be called *non-differentiable Toda lattice* (NDTL)). Accordingly (Jackson, 1992; Chua & Yang, 1988a; Chua & Yang, 1988b) by mapping there result two cellular neural networks, one of them being specific to the differentiable scale (we shall call it *differentiable cellular neural network* (DNNC)) and the other one being specific to the non-diferentiabile scale (will be called *non-differentiable cellular neural network* (NDNNC)). These two networks, even they have distinct functionalities, one of them being associated to "the outer" of the complex system through the diferentiable scale and the other one being associated to the "inner" of the same complex system through the non-diferentiable scale, simultaneously coexist and are interdependent. Practically, together with the differentiable-non-differentiable Toda lattice pair, the differentiable-non-differentiable cellular neural network pair is generated by mapping.

The self similarity of the space-time cnoidal oscillation modes specifies the existence of some "cloning" mechanisms (full and fractional wave function revivals - a wave functions evolves in time to a state describable as a collection of spatially distributed sub-wave functions that each closely reproduces the initial wave function shape (Aronstein & Stroud, 1997)). All these show a direct connection between the non-differentiable structure of the "dynamics regimes" and holographic principle (in any finite volume of a complex system it finds its entire structure) (Mandelbrot, 1983; Nottale, 2011).

In our opinion the previous mechanisms of the human brain play a fundamental role in the bacterial growth process (under external constraints such as injuries the adequate "software" in the brain comes into motion, thus activating specific bacteria. We note that "Bacterial growth is the asexual reproduction or cell division, of a bacterium into two daughter cells, in a process called binary fission. Providing no mutational event occurs the resulting daughter cells are genetically identical to the original cell. Hence local doubling of the bacterial population occurs. Both daughter cells from the division do not necessarily survive. However, if the number surviving exceeds unity on average, the bacterial population undergoes exponential growth. The measurement of an exponential bacterial growth curve in batch culture was traditionally a part of the training of all microbiologists; the basic means requires bacterial enumeration (cell counting) by direct and individual (microscopic, flow cytometry), direct and bulk (biomass), indirect and individual (colony counting), or indirect and bulk (most probable number, turbidity, nutrient uptake) methods. Models reconcile theory with the measurements" (Skarstad *et al*. 1983; Zwietering *et al*. 1990; https://en.wikipedia.org/wiki/Bacterial_growth#cite_notepmid6341358-1).

3. Conclusions

The main conclusions of the present paper are the following:

i) In the ESRT, the Schrödinger type geodesics and non-differentiable hydrodynamic equations are obtained;

ii) In the one-dimensional case, a solution of the non-differentiable hydrodynamic equations in the form of cnoidal oscillation modes is given;

iii) Associating a one-dimensional Toda lattice to the cnoidal oscillation modes, two cellular neural networks result by mapping, one of them being specific to the differentiable scale (differentiable cellular neural network (DNNC)) and the other one being specific to the non-differentiable scale (non-differentiable cellular neural network (NDNNC));

iv) Specific mechanisms required by the cellular neural network of the brain in the bacterial growth process are analyzed.

Implications of these cellular neural networks in the functionality of some biological systems are given in Duceac *et al*., 2015a and Duceac *et al*., 2015b.

REFERENCES

Armitage J.V., *Elliptic Functions*, Cambridge University Press, Cambridge, 2006.

Aronstein D.R., Stroud C.L., *Fractional Wave-Function Revivals in the Infinite Square Well*, Physical Review A, **55**, *6*, 4526−4537, 1997.

Chow T.W.S., *Neural Networks and Computing,* Learning Algorithms and Applications, Series in Electrical and Computer Engineering, 2007.

Chua L.O., Yang L., *Cellular Neural Networks: Applications,* IEEE Transactions on Circuits and Systems, **35**, *10*, 1273−1290, 1988a.

Chua L.O., Yang L., *Cellular Neural Networks: Theory,* IEEE Transactions on Circuits and Systems, **35**, *10*, 1257−1272, 1988b.

Duceac L.D., Nica I., Gaţu I., *Fractal Bacterial Growth,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 57−64, 2015a.
Duceac L.D., Nica I., Iacob D.D., *Chaos and Self-Structuring in Cellular Growth Dynamics,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 65−72, 2015b.
Flake G.W., *The Computational Beauty of Nature,* MIT Press, Cambridge, MA, 1998.
Gottlieb I., Agop M., Jarcău M., *El Naschie's Cantorian Space-Time and General Relativity by Means of Barbilian's Group. A Cantorian Fractal Axiomatic Model of Space-Time,* Chaos, Solitons and Fractals, **19**, *4*, 705−730, 2004.
Gurlui S., Agop M., Strat M., Băcăiţă S., Cerepaniuc A., *Some Experimental and Theoretical Results on the on the Anodic Patterns in Plasma Discharge,* Physics of Plasmas, **13**, I, 2006.
https://en.wikipedia.org/wiki/Bacterial_growth#cite_notepmid6341358-1
Jackson E.A., *Perspectives on Nonlinear Dynamics,* Cambridge University Press, Cambridge, 1992.
Karayiannis N.B., Venetsanopoulos A.N., *Artificial Neural Networks. Learning Algorithms, Performance Evaluation, and Applications,* The Springer International Series in Engineering and Computer Science, **209**, 1993.
Mandelbrot B., *The Fractal Geometry of Nature,* Updated and Augm. Ed., W.H. Freeman, New York, 1983.
Mitchell M., *Complexity: A Guided Tour,* Oxford University Press, Oxford, 2009.
Nottale L., *Scale Relativity and Fractal Space -Time - A New Approach to Unifying Relativity and Quantum Mechanics,* Imperial College Press, London, 2011.
Phillips A.C., *Introduction to Quantum Mechanics,* Wiley, New York, 2003.
Skarstad K., Steen H.B., Boye E., *Cell Cycle Parameters of Slowly Growing Escherichia Coli B/r Studied by Flow Cytometry,* J. Bacteriol., ***154***, *2,* 656−662, PMC 217513, PMID 6341358 (1983).
Timofte A., Casian-Botez I., Scurtu D., Agop M., *System Dynamics Control Through the Fractal Potential,* Acta Physica Polonica A, **119**, *3*, 304−311 (2011).
Toda M., *Theory of Nonlinear Lattices,* New York: Springer Verlag, 1981.
Zwietering M.H., Jongenburger I., Rombouts F.M., van 'T Riet K., *Modeling of the Bacterial Growth Curve,* Applied and Environmental Microbiology, **56**, *6*, 1875−1881, PMC 184525, PMID 16348228 (1990).

REŢELE NEURALE CELULARE DIFERENŢIABILE ŞI NEDIFERENŢIABILE CU IMPLICAŢII ÎN PROCESUL DE CREŞTERE A BACTERIILOR. UN MODEL MATEMATIC (I)

(Rezumat)

În cadrul Teoriei Extinse a Relativităţii de Scală, sunt obţinute geodezicele de tip Schrödinger şi cele ale modelului hidrodinamic. În cazul unidimensional al modelului hidrodinamic nediferenţiabil, este obţinută o soluţie analitică sub forma modurilor de oscilaţie cnoidale. Asociind modurilor de oscilaţii cnoidale o reţea de tip Toda, prin mapare, rezultă două tipuri de reţele celulare neurale, o reţea celulară neurală diferenţiabilă şi o reţea celulară neurală nediferenţiabilă. Câteva implicaţii ale modelului în mecanismul de creştere al bacteriilor este de asemenea considerat în cadrul lucrării.

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 62 (66), Numărul 1, 2016
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

DIFFERENTIABLE AND NON-DIFFERENTIABLE CELLULAR NEURAL NETWORKS WITH IMPLICATIONS IN THE BACTERIAL GROWTH PROCESS. PROPERTIES (II)

BY

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Received: January 11, 2016
Accepted for publication: April 18, 2016

Abstract. Properties of cellular neural networks (differentiable and non-differentiable cellular neural networks) are given: wave number spectrum, phase velocity spectrum, quasi-period spectrum etc. In such perspective, the conditions of non-differentiable-non-differentiable cellular neural networks coherence are given and this could explain not only the way in which information is stored and transmitted in the brain but also the way in which the communications codes can be generated. Some implications of the model in the growth bacterial process are established.

Keywords: differentiable cellular neural network; non-differentiable cellular neural network; coherence; information; communication codes.

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1. Introduction

According to Gavriluț *et al*., 2016, two types of cellular neural networks (differentiable and non-differentiable cellular neural networks) are obtained. These cellular neural networks have some specific properties that will be highlighted in this paper.

2. Results and Discussions

2.1. Specific Properties of the Cellular Neural Networks

The Toda lattices at differentiable and non-differentiable scales and by mapping the two corresponding associated cellular networks have some following characteristic parameters:

i) Wave number spectrum

$$k(s)=\frac{\pi a^{1/2}}{sK(s)},k'(s')=\frac{\pi a^{1/2}}{s'K(s')} \tag{1}$$

ii) Phase velocity spectrum

$$U_p(s)=6\bar{N}+4a\left[\frac{3E(s)}{K(s)}-\frac{1+s^2}{s^2}\right],U'_p(s')=6\bar{N}+4a\left[\frac{3E(s')}{K(s')}-\frac{1+s'^2}{s'^2}\right] \tag{2}$$

iii) Quasi-period spectrum

$$T(s)=\frac{1}{\frac{3\pi\bar{N}a^{1/2}}{sK(s)}+\frac{2a^{3/2}}{sK(s)}\left[\frac{3E(s)}{K(s)}-\frac{1+s^2}{s^2}\right]}$$

$$T(s')=\frac{1}{\frac{3\pi\bar{N}a^{1/2}}{s'K(s')}+\frac{2a^{3/2}}{s'K(s')}\left[\frac{3E(s')}{K(s')}-\frac{1+s'^2}{s'^2}\right]} \tag{3}$$

The significances of the quantities from (1)−(3) are given in Gavriluț *et al*., 2016.

Expliciting the oscillation modulus of the Toda network and the functionality of the associated cellular neural network at differentiable scale is achieved by the following degenerations of the elliptic function cn in the modulus s (Armitage, 2006).

i) For $s \to 0,$ the solution (19) from the paper of Gavriluț *et al*., 2016 reduces to the harmonic package type sequence:

$$N = \langle N \rangle + a\cos\left(\bar{\theta}\right),\ \langle N \rangle = \bar{N} + a \tag{4}$$

which is described by the wave number spectrum

$$k \cong \frac{2a^{1/2}}{s} \tag{5}$$

the phase velocity spectrum:

$$U_p \cong 6\bar{N} + 8a - k^2 \tag{6}$$

and the quasi-period spectrum

$$T \cong \frac{2\pi}{6\bar{N}k + 8ak - k^3} \tag{7}$$

ii) For $s \to 1,$ the solution (19) from Gavriluț *et al*., 2016 reduces to a soliton package type sequence:

$$N \cong \bar{N} + a_1 \sec h^2 \left[\left(\frac{a_1}{6} \right)^{1/2} \left(\theta - \theta_0 \right) \right] \tag{8}$$

characterized by the wave number spectrum

$$\Lambda \cong \frac{(2a_1)^{1/2}}{4k_1}, a_1 = 2a, k_1 = \frac{k}{2\pi} \tag{9}$$

the phase velocity spectrum

$$u_p \cong 6\bar{N} + 2a_1 - 12{k_1}^2 a_1^{1/2} \tag{10}$$

and the quasi-period spectrum

$$T \cong \frac{1}{6\bar{N}k_1 + 2a_1k_1 - 12{k_1}^2 a_1^{1/2}} \tag{11}$$

For $s = 0,$ the solution (19) from Gavriluț *et al*., 2016 reduces to an harmonic type sequence, while for $s = 1$ it reduces to a soliton type one. For details on the degeneration of elliptic function *cn* see Armitage, 2006, while for details related to nonlinear solutions (soliton, soliton package etc.) (Jackson, 1992).

As shown before, nonlinear waves have long been an interest to scientists in a variety of disciplines. A multitude of applications have been employed, ranging from fluid dynamics and plasma physics to even neuroscience and biology. It is with this same interest that we have chosen the topic of computational waves. Waves are omnipresent, from tsunamis in the ocean, to gamma waves and sonic booms. In fact, there are currently several wave pulses propagating through the neurons in our brain due to the firing of action potentials.

In the physics literature, the terms soliton and solitary wave are often used interchangeably. Solitary waves (and solitons) arise in both continuous systems such as the KdV equation and discrete systems such as the Toda lattice (Toda, 1981; Toda, 1983) and in both one and multiple spatial dimensions. Key issues in studying solitary waves also include linear versus nonlinear (of course), integrable versus nonintegrable, persistent versus transient, asymptotics (*i.e.*, consideration of time scales), localization in physical space versus Fourier space, and the effects of noise.

The important property of the Toda equation is the existence of so called soliton solutions, that is, pulslike waves which spread in time without changing their size or shape and interact with each other in a particle-like way. This is a surprising phenomenon, since for a generic linear equation one would expect spreading of waves (dispersion) and for a generic nonlinear force one would expect that solutions only exist for a finite time (breaking of waves). Obviously, our particular force is such that both phenomena cancel each other giving rise to a stable wave existing for all time (Jackson, 1992).

Simultaneously, it is also achieved the explanation at non-differentiable scale of the Toda network oscillation modes and of the associated cellular neural network functionalities through the elliptic functions *cn* degenerations in the complementary modulus s'.

Eliminating the amplitude a between (1) and (2) we obtain the following dispersion equations:

$$\left(U_p - 6\bar{N}\right)\bar{\lambda}^2 = 16A(s), k = \frac{2\pi}{\bar{\lambda}},$$

$$\left(U'_p - 6\bar{N}\right)\overline{\lambda'}^2 = 16A'(s'), k' = \frac{2}{\overline{\lambda'}} \tag{12}$$

where

$$A(s) = 3s^2 K(s) E(s) - \left(1+s^2\right) K^2(s)$$

$$A'(s') = 3s'^2 K(s') E(s') - \left(1+s'^2\right) K'^2(s') \tag{13}$$

Nonlinearity s generates two distinct dynamics regimes of the Toda lattice respectively of the cellular neural network associated to the differentiable scale: non-quasi-autonomous regimes (by harmonic type sequences, harmonic package type sequences) and quasi-autonomous regimes (by soliton type sequences, soliton package type sequences) respectively. The dependency *A*(*s*) - see Fig. 2 a,b, specifies that the value $s \cong 0.7$ separates the two dynamic regimes. For $0 \le s \le 0.7$, *i.e.*, in non-quasi-autonomous regime, the variable $A(s) \cong$ const., situation in which the first of eqs. (12) takes the form:

$$\left(U_p - 6\bar{N}\right)\lambda^2 \cong \text{const.} \tag{14}$$

while for $0.7 \le s \le 1$, *i.e.*, in quasi-autonomous regime, the relation (14) loses its validity. Simultaneously, the non-linearity s' generates two distinct dynamics regimes of the Toda lattice respectively of the cellular neural network associated to the non-differentiable scale.

2.2. The Coherence of Differentiable-non-Differentiable Cellular Neural Network

According to our previous results, differentiable-non-differentiable Toda lattice pair is mathematically described by the elliptic function cn^2. Now, invoking a fundamental theorem from elliptic functions theory, it is well known that two elliptic functions are equivalent if and only if there exists an homographic transformation between the ratio of their periods (Armitage, 2006):

$$\mu' = \frac{a\mu + b}{c\mu + d}, a, b, c, d \in N \tag{15}$$

where μ' and μ define the ratios of the these elliptic functions periods. Moreover, if such theorem is fulfilled, then there exist an algebraic relation between these two elliptic functions.

In such a framework, we shall interpret elliptic function equivalence theorem as a coherence condition of differentiable-non-differentiable Toda lattice pairs and, moreover, its consequence, as a "communication" ("relationship") way of pairs.

Now, by mapping, the previous results imply that differentiable-non-differentiable cellular neural network pairs are coherent if and only if the condition (15) is fulfilled. Moreover, if this condition is fulfilled, then the pairs can "store" and "transmit" information in a specific algebraic language.

3. Conclusions

Assuming that the nervous impulse transmission through brain's neuronal network is achieved on continuous and non-differentiable curves, the hydrodynamic version of scale relativity in arbitrary constant fractal dimension is presented (non-differentiable hydrodynamics) in Gavriluț *et al*., 2016. In such context, the one-dimensional solution of the non-differentiable hydrodynamic model for states density is obtained in the form of spatio-temporal cnoidal modes, assuming that the external scalar potential is proportional with the states density. Moreover, the spatio-temporal cnoidal modes are asssimilated to a one-dimensional Toda network and hence, by mapping, to a cellular neural network.

In the present paper, we give some specific properties of the two types of cellular neural networks. Then the following main conclusions result:

i) Cnoidal modes double periodicity induces differentiable-non-differentiable Toda pair and hence, by mapping, differentiable-non-differentiable cellular neural network pair. Pairs components are simultaneously generated, are interdependent and are characterized by some parameters such as wave number spectrum, phase velocity spectrum and quasi-period spectrum;

ii) Each of pair's components presents two functionality regimes, one of them being induced by the harmonic waves and harmonic wave packages and the other one by solitons and packages of solitons;

iii) The dispersion equations are obtained for each of the two pair's components;

iv) Pairs' coeherence imposes "storage" and "transmission" of informations in the form of a specific algebraic language.

Unlike the electronic calculator which has a hard structure divided by artificial algorithms, the spectral component corresponding to the hardware particles, has the same artificial behaviour, without having the fractality of the natural development.

As a result, there is no consistency between the cellular network of the substance and the spectral undulatory one. The development of the neural network is performed according to fractal criteria, the same as for all the other parts and systems of the human body. Consequently, the spectrum field created by the undulation of the particles of the neural network is consistent, allowing the information to be processed inside the neural network and also in the spectrum field (Hilbert space), where there are the aspatial and atemporal components which enable the memory and also the complex component that allows the possibility of multidimensional processing which can explain the superior psychic processes, the conceptualization, the semantics, the abstract etc.

Therefore, at any scale there are the two types of realities that coexist, the differential and non-differential, highlighted by the hydrodynamic theory and the stochastic one. Another major difference between the electronic calculator and the human brain is done by the analogical feature of psychic processing, unlike the digital signal processing. Analog signal processing is enhanced by the topology configuration character of the processing, being not only a numeric processing but also one determined by the geometric topology.

The dimensional dynamics, from 0 to infinite, which is realized in our reality up to 3 dimensions, can be performed multidimensionally in the complex field of psychic reality (through the fractal potential).

In our opinion the previous mechanisms of the human brain play a fundamental role in the growth bacterial process. Once the cellular neural network is activated, specific communcation codes are also employed by the relevant celles.

Some implications of cellular neural networks properties in the functionality of biological systems are given also in the references (Duceac *et al*., 2015a; Duceac *et al*., 2015b).

Finally, we give an example of how the brain controls, by means of specific softwares, the bacterial growth processes. “The genus *Mycobacterium* is best known for its two major pathogenic species, *M. tuberculosis* and *M. leprae*, the causative agents of two of the world's oldest diseases, tuberculosis and leprosy, respectively. *M. tuberculosis* kills approximately two million people each year and is thought to latently infect one-third of the world's population. One of the most remarkable features of the nonsporulating *M. tuberculosis* is its ability to remain dormant within an individual for decades before reactivating into active tuberculosis. Thus, control of cell division is a critical part of the disease. The mycobacterial cell wall has unique characteristics and is impermeable to a number of compounds, a feature in part responsible for inherent resistance to numerous drugs. The complexity of the cell wall represents a challenge to the organism, requiring specialized mechanisms to allow cell division to occur. Besides these mycobacterial specializations, all bacteria face some common challenges when they divide. First, they must maintain their normal architecture during and after cell division. In the case of mycobacteria, that means synthesizing the many layers of complex cell wall and maintaining their rod shape. Second, they need to coordinate synthesis and breakdown of cell wall components to maintain integrity throughout division. Finally, they need to regulate cell division in response to environmental stimuli” (Hett & Rubin, 2008).

Moreover in the same work (Hett & Rubin, 2008) “discuss these challenges and the mechanisms that mycobacteria employ to meet them. Because these organisms are difficult to study, in many cases we extrapolate from information known for gram-negative bacteria or more closely related GC-rich gram-positive organisms”.

REFERENCES

Armitage J.V., *Elliptic Functions,* Cambridge University Press, Cambridge, 2006.
Duceac L.D., Nica I., Gațu I., *Fractal Bacterial Growth,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 57−64, 2015a.
Duceac L.D., Nica I., Iacob D.D., *Chaos and Self-Structuring in Cellular Growth Dynamics,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 65−72, 2015b.
Gavriluț G., Crumpei G., Butuc I., Duceac L.D., *Differentiable and Non-Differentiable Cellular Neural Networks with Implications in the Bacterial Growth Process, A Mathematical Model (I)*. Bul. Inst. Polit. Iaşi, Vol. 62 (66), No. 1, pp. 67−76, 2016 (in press).
Hett E.C., Rubin E.J., *Bacterial Growth and Cell Division: a Mycobacterial Perspective,* Microbiol. Mol. Biol. Rev., March 2008, **72**, *1*, 126−156, doi: 10.1128/MMBR.00028-07.
Jackson E.A., *Perspectives on Nonlinear Dynamics,* Cambridge University Press, Cambridge, 1992.
Toda M., *Theory of Nonlinear Lattices,* New York: Springer Verlag, 1981.
Toda M., *Nonlinear Lattice and Soliton Theory,* IEEE Trans. CAS, **30**, 542−554, 1983.

REȚELE NEURALE CELULARE DIFERENȚIABILE ŞI NEDIFERENȚIABILE CU IMPLICAȚII ÎN PROCESUL DE CREŞTERE A BACTERIILOR. PROPRIETĂȚI (II)

(Rezumat)

Sunt prezentate proprietăți ale rețelelor celulare neurale (rețele celulare neurale diferențiable şi nediferențiabile): spectrul numărului de undă, spectrul vitezei de fază, spectrul cvasi-perioadei etc. Din această perspectivă sunt precizate condițiile pentru coerența rețelelor celulare neurale diferențiabile şi nediferențiabile, ceea ce poate explica nu numai modul în care se stochează şi transmite informația, ci şi modul în care pot fi generate codurile de comunicare. Sunt prezentate câteva exemple prin care codurile de comunicare intervin în procesele de creştere celulară.

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 62 (66), Numărul 1, 2016
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

ON A SPECIAL FRACTAL CELLULAR NEURAL NETWORK BY MEANS OF FRACTAL POTENTIAL AND ITS IMPLICATIONS IN THE BACTERIAL GROWTH PROCESS

BY

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Received: January 11, 2016
Accepted for publication: April 18, 2016

Abstract. Considering that nervous impulse transmission in the brain neuronal network is achieved on continuous but non-differentiable curves (fractal curves), in the scale relativity hydrodynamic variant (with constant arbitrary fractal dimension) a special cellular neural network class at fractal scale is introduced. Thus, this solution with infinite fractal "energy" implies through a fractal potential cnoidal oscillation modes of the fractal states density. These modes can be associated with an one-dimensional Toda fractal lattice and, by mapping, with a fractal cellular neural network. The solution with finite fractal "energy" generated by a spontaneous symmetry breaking mechanism induces elements of a fractal logic as fractal bit, fractal gates etc. The implication of the model in the bacterial growth process is given.

Keywords: fractal potential; fractal kink solution; symmetry breaking; Toda lattice; cellular neural network.

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1. Introduction

In recent papers (Gavriluț *et al*., 2016; Butuc *et al*., 2016) it is proved that in the fractal approximation of the motion, one can introduce two new concepts of cellular neural networks, precisely, the differentiable cellular neural network and the non-differentiable cellular neural one. Their coherence assures not only the functional-structural compatibility of any living structure, but also their comunication code.

In this paper we introduce a new class of cellular neural networks, based on the fractal potential. In such context, the spontaneous symmetry breaking will generate patterns. Based on their functionality, the specific communication code will be induced.

2. Results and Discussions

2.1. The Mathematical Model

Let us consider the fractal hydrodynamic equations (Colotin *et al*., 2009; Timofte *et al*., 2011). Then for the fractal static states, we have

$$\partial_t = 0, \boldsymbol{V}_D = D\left(dt\right)^{(2/D_F)-1} \nabla \mathrm{S} = 0 \tag{1}$$

In phase coherence S = const. of the structural units (Mitchell, 2009) the fractal hydrodynamic equations become with the substitutions

$$U = E\rho, E = \text{const.} > 0, \rho^{1/2} = f, \mathbf{D} = D\left(dt\right)^{(2/D_F)-1} \tag{2}$$

become

$$\frac{2m_0\mathbf{D}^2}{E}\Delta f = f^3 - f \tag{3}$$

Particularly, in the one-dimensional case $\xi = x(E/2m_0\mathbf{D}^2)^{1/2}$, (3) takes the form:

$$\partial_{\xi\xi} f = f^3 - f \tag{4}$$

One can refer to Timofte *et al*., 2011 for the significance of the quantities from (1)-(4).

The eq. (4) can be also obtained by means of the fractal variational principle $\delta\int L d\tau = 0$, with $d\tau$ the fractal elementary volume applied to the fractal Lagrangean density:

$$L = \frac{1}{2}(\partial_\xi f)^2 - \wp\left(f\right) \tag{5}$$

with the fractal potential

$$\wp(f)=\left(\frac{f^4}{4}\right)-\left(\frac{f^2}{2}\right) \tag{6}$$

The eq. $\partial_{\xi\xi} f = 0$ has the solutions $f_F^{(1)} = 0, f_F^{(2,3)} = \pm 1$. By calculating the second derivative with respect to ξ of the fractal "potential" (6) and substituting the values $f^{(1,2,3)}$ into the result of this differentiation we find $\wp_{\xi\xi}(0) = -1, \wp_{\xi\xi}(\pm 1) = 2 > 0$, *i.e.*, the solution $f_F^{(2,3)} = \pm 1$ is associated with the minimum fractal "energy". Hence, the model under consideration has a double degenerated vacuum fractal state.

From (5) it results both fractal "energy",

$$\varepsilon(f)=\int_{-\infty}^{\infty} d\xi \frac{1}{2}\left[(\partial_{\xi} f)^2+\wp(f)\right] \tag{7}$$

and the fractal "energy" relative to the fractal vacuum

$$\varepsilon(f)-\varepsilon(f_F)=\int_{-\infty}^{\infty} d\xi \frac{1}{2}\left[\left(\partial_{\xi} f\right)^2+\frac{1}{4}\left(f^2-1\right)^2\right] \tag{8}$$

Since all terms in (8) are positive and in view of the infinite limits of integration, the finiteness of the fractal "energy" implies that at $\xi \to \pm\infty$

$$\partial_{\xi} f = 0, \frac{1}{4}\left(f^2-1\right)^2 = 0 \tag{9}$$

From this, it follows that at $\xi \to \pm\infty$, the function $f(\xi)$ tends to its fractal vacuum value $f_F^{(2,3)} \to \pm 1$. In order to find the explicit form of the solution of (4), we multiply it by $\partial_{\xi} f$ and subsequently over ξ. This yields

$$\frac{1}{2}(\partial_{\xi} f)^2 = -\frac{f^2}{2}+\frac{f^4}{4}+\frac{1}{2}f_0 \tag{10}$$

where f_0 is an integration constant. From this, we have:

$$\xi-\xi^0=\int_0^f \frac{df}{\sqrt{\frac{f^4}{4}-\frac{f^2}{2}+\frac{1}{2}f_0}} \tag{11}$$

where ξ^0 is the other integration constant. To this solution it corresponds for an arbitrary f_0, an infinite value of the fractal "energy" $\varepsilon(f)$. To obtain the solution with finite fractal "energy", we make use of the boundary conditions $f_F^{(2,3)} = \pm 1$. From (10) it results that $f_0 = \frac{1}{2}$. Replacing this value of f_0 into (18), the solution $f_k(\xi)$ of the field eq. (10) with a finite fractal "energy" is

$$f_k(\xi) = f\left(\xi - \xi^0\right) = \tanh\left[\frac{1}{\sqrt{2}}\left(\xi - \xi^0\right)\right] \tag{12}$$

This is called the *fractal kink solution*. Combining (8) with the expression $f_F^{(2)} = 1$ and the expression for f_k, we obtain the fractal "energy" of the fractal kink relative to the fractal vacuum:

$$\varepsilon\left(f_k\right) - \varepsilon\left(f_F\right) = \frac{2\sqrt{3}}{3} \quad . \tag{13}$$

The fractal kink solution is obtained by a fractal spontaneous symmetry breaking (the fractal vacuum state is not invariant with respect to the fractal group of transformations which makes invariant eq. (4), while the fractal lagrangean is invariant with respect to the same group). This corresponds to a fractal pattern.

2.2. Fractal Topology and Logic

A fractal topological method can be applied because the admissible number of fractal kinks is determined by the fractal topological properties of the fractal symmetry group of eq. (4). In this context, the following problems must be solved:

i) The number of admissible fractal kink solutions determined by the fractal topological properties of the eq. (4);

ii) *The fractal topological charge.*

The fractal kink solution can be obtained as mapping of a fractal spatial zero-sphere S^0, taken at infinity onto the fractal vacuum manifold of the model (4). The fractal homotopy group for this model is $\Pi_0(Z_0) = Z_2$, *i.e.*, the model gives rise to two solutions: a constant solution and the fractal kink solution. Details on an usual homotopic mapping are given in Jackson, 1992.

The associated fractal topological charge is:

$$q = \frac{1}{2}\int_{-\infty}^{\infty} j(\xi)d\xi = \frac{1}{2}\int_{-\infty}^{\infty} \frac{df}{d\xi}d\xi = \frac{1}{2}\left[f\left(+\infty\right) - f\left(-\infty\right)\right] \tag{14}$$

The fractal vacuum solution (absence of spatial gradients) and the fractal kink solution can be characterized by attributing the $q = 0$ and $q = 1$, respectively (the result is obtained by an adequate normalization f). Since (4) is a fractal Ginzburg-Landau equation type, it follows that $q=0$ and the fractal vacuum solution describes the behavior of the fractal fluid in the absence of self-structuring, *i.e.*, its fractal ground states, while $q=1$ and the fractal kink solution describes the behavior of the fractal fluid in the presence of self-structuring, *i.e.*, the fractal pattern.

Now, one can associate to these values of the fractal topological charge, the fractal bit, that is, a fractal physical system which can exist in two distinct fractal states (an unstructured state or of fractal vacuum and a structured one or of fractal superconductivity). These states are used in order to represent $0(dt)$ and $1(dt)$, that is a single binary fractal digit. The only possible operations (fractal gates) that are compatible with such systems are

the fractal IDENTITY

$$0(dt)\to 0(dt), 1(dt)\to 1(dt). \tag{15}$$

and

the fractal NOT (FNOT)

$$0(dt)\to 1(dt), 1(dt)\to 0(dt) \tag{16}$$

Therefore, unlike the standard bit, the fractal bit is a fractal system with two self-similar and scale independent states.

2.3. Cnoidal Oscillation Modes, Toda Lattices and Cellular Neural Networks by Means of Fractal Potential

Based on the method from Jackson (1992), the solution with infinite fractal energy is obtained by inversion of the elliptic integral (18) and has the expression

$$f=\left(\frac{2s^2}{1+s^2}\right)^{1/2} sn\left[\frac{\xi-\xi^0}{\left(1+s^2\right)^{1/2}};s\right] \tag{17}$$

where sn is Jacobi elliptic function of modulus s (Armitage, 2006).

By the degeneracy of the elliptic function sn with respect to its modulus s one obtains for $s\to 1$ a fractal harmonic waves package and fractal harmonic waves for $s=0$, while for $s\to 1$ is obtained a fractal solitons package and for $s=1$, the fractal kink (12).

In such a frame, the normalized fractal potential in the form

$$\bar{Q}=-\frac{1}{f}\frac{d^2 f}{d\xi^2}=1-f^2=\frac{1-s^2}{1+s^2}+\frac{2s^2}{1+s^2}cn^2\left[\frac{\xi-\xi^0}{\left(1+s^2\right)^{1/2}};s\right] \tag{18}$$

where cn is Jacobi elliptic function of modulus s (Armitage, 2006) specifies the fact that the interactions between fractal fluid entities are achieved through cnoidal oscillation modes of the fractal states density. Since these oscillation modes are associated to an one-dimensional Toda fractal lattice (Toda, 1981), then in its functionality we shall distinguish different regimes induced by those of a fractal "fluid" flow: the fractal non-quasi-autonomous given by fractal harmonic waves and fractal harmonic waves packages, the fractal quasi-autonomous regime given by fractal solitons and fractal soliton packages and the fractal transitory one given by fractal mixtures of fractal harmonic waves package - solitons package type etc. For the standard details see Whitman, 1974.

Now, by mapping, the one-dimensional Toda fractal lattice implies a special fractal cellular neural network (CNN) class with her entire soft, "archeology" etc. For the standard details see Drazin, 1992, Jackson, 1992.

It should be noticed the fact that all above mentioned consequences are coordinated by the fractal potential.

3. Conclusions

The main conclusions of the present paper are the following:

i) Assuming that the nervous impulse transmission through brain's neuronal network is achieved on continuous and non-differentiable curves, the hydrodynamic version of scale relativity in arbitrary constant fractal dimension is used (fractal hydrodynamics);

ii) Assuming that the external scalar potential is proportional with the fractal states density, the one-dimensional solution with finite fractal "energy" in the form of fractal kink is obtained. This solution breaks the fractal vacuum symmetry and generates fractal patterns by means of spontaneous symmetry breaking mechanism (these patterns can be identified with bacteria). Then, the phase coherence of the fractal patterns will produce a self-structuring of the fractal vacuum which is interpreted as a tendancy of the complex system to make structures (patterns);

iii) Since the admissible number of fractal kinks is determined by the fractal topological properties of the symmetry group of eq. (4), a topological fractal method can be applied. Then, some elements of a fractal logic as fractal bit, fractal gates (fractal IDENTITY, fractal NOT) etc. are obtained;

iv) The solution with infinite fractal "energy" is given. Then, by mes of specific fractal potential, the interactions among fractal "fluids" entities are achieved through cnoidal oscillation modes of the fractal states. Since these oscillation modes are associated to a one-dimensional Toda lattice, in its functionality we shall distinguish three various regimes induced by those of a fractal fluid flow: fractal non-quasi-autonomous regime through fractal harmonic waves and fractal harmonic wave packages, the quasi-autonomous regime through fractal soliton and fractal soliton packages and the transitory one through fractal mixtures of harmonic wave package - solitons package type. Now, by mapping, the one-dimensional Toda lattice can be associated with a special fractal cellular neural network class and hence the entire arsenal implied.

Some implications of the model we discussed in the present paper regarding the functionality of biological systems are given in Stoica *et al*., 2015; Duceac *et al*., 2015a; Duceac *et al*., 2015b; Doroftei *et al*., 2016; Nemeş *et al*., 2015; Postolache *et al*., 2016; Ştefan *et al*., 2016.

In the following we give an example on the role communication codes play in the bacterial growth process. "In recent years it has become clear that the production of N-acyl homoserine lactones (N-AHLs) is widespread in Gram-negative bacteria. These molecules act as diffusible chemical communication signals (bacterial pheromones) which regulate diverse physiological processes including bioluminescence, antibiotic production, plasmid conjugal transfer and synthesis of exoenzyme virulence factors in plant and animal pathogens. The paradigm for N-AHL production is in the bioluminescence (lux) phenotype of Photobacterium fischeri (formerly classified as Vibrio fischeri) where the signaling molecule N-(3-oxohexanoyl)-L-homoserine lactone (OHHL) is synthesized by the action of the LuxI protein. OHHL is thought to bind to the LuxR protein, allowing it to act as a positive transcriptional activator in an autoinduction process that physiologically couples cell density (and growth phase) to the expression of the bioluminescence genes. Based on the growing information on LuxI and LuxR homologues in other N-AHL-producing bacterial species such as Erwinia carotovora, Pseudomonas aeruginosa, Yersinia enterocolitica, Agrobacterium tumefaciens and Rhizobium leguminosarum, it seems that analogues of the P. fischeri lux autoinducer sensing system are widely distributed in bacteria. The general physiological function of these simple chemical signaling systems appears to be the modulation of discrete and diverse metabolic processes in concert with cell density. In an evolutionary sense, the elaboration and action of these bacterial pheromones can be viewed as an example of multicellularity in prokaryotic populations" (Salmond *et al.*, 1995).

REFERENCES

Armitage J.V., *Elliptic functions,* Cambridge University Press, Cambridge, 2006.

Butuc I., Gavriluț A., Gavriluț G., Duceac L.D., *Differentiable and Non-Differentiable Cellular Neural Networks with Implications in the Bacterial Growth Process. Properties (II),* Bul. Inst. Polit. Iaşi, Vol. 62 (66), No. 1, pp. 77−84, 2016 (in press).

Colotin M., Pompilian G.O., Nica P., Gurlui S., Păun V., Agop M., *Fractal Transport Phenomena Through the Scale Relativity Model,* Acta Physica Polonica A, **116**, *2*, 157−164, 2009.

Doroftei B., Duceac L.D., Iacob D.D., Dănilă N., Volovăţ S., Scripcariu V., Agop M., Aursulesei V., *The Harmonic Oscillator Problem in the Scale Relativity Theory. Its Implications in the Morphogenesis of Structures at Various Scale Resolution,* Journal of Computational and Theoretical Nanoscience, 2016 (in press).

Drazin P.G., *Nonlinear Systems,* Cambridge University Press, Cambridge UK, 1992.

Duceac L.D., Nica I., Gațu I., *Fractal Bacterial Growth,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 57−64, 2015a.

Duceac L.D., Nica I., Iacob D.D., *Chaos and Self-Structuring in Cellular Growth Dynamics,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 65−72, 2015b.

Gavriluț G., Crumpei G., Butuc I., Duceac L.D., *Differentiable and Non-Differentiable Cellular Neural Networks with Implications in the Bacterial Growth Process. A Mathematical Model (I),* Bul. Inst. Polit. Iaşi, Vol. 62 (66), No. 1, pp. 67−76, 2016 (in press).

Jackson E.A., *Perspectives on Nonlinear Dynamics*. Cambridge University Press, Cambridge, 1992.

Mitchell M., *Complexity: A Guided Tour,* Oxford University Press, Oxford, 2009.

Nemeş R.M., Duceac L.D., Vasincu E.G., Agop M., Postolache P., *On the Implications of the Biological Systems Fractal Morpho-Functional Structure,* U.P.B. Sci. Bull., Series A, **77**, *4*, 263−272, 2015.

Postolache P., Duceac L.D., Vasincu E.G., Agop M., Nemeş R.M., *Chaos and Self-Structuring Behaviors in Lung Airways,* U.P.B. Sci. Bull., **6**, 2016 (in press).

Salmond G.P., Bycroft B.W., Stewart G.S., Williams P., *The Bacterial 'Enigma': Cracking the Code of Cell-Cell Communication,* Mol. Microbiol., 1995 May; **16**, *4*, 615−624.

Ştefan G., Duceac L.D., Gațu I., Păun V.P., Agop M., Aursulesei V., Manea L.R., Rotaru M., *Structures Morphogenesis in Complex Systems at Nanoscale,* Journal of Computational and Theretical Nanoscience, 2016 (in press).

Stoica C., Duceac L.D., Lupu D., Boicu M., Mihăileanu D., *On the Fractal Bit in Biological Systems*, Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 47−55, 2015.

Timofte A., Casian-Botez I., Scurtu D., Agop M., *System Dynamics Control Through the Fractal Potential,* Acta Physica Polonica A, **119**, *3*, 304−311 (2011).

Toda M., *Theory of Nonlinear Lattices,* New York: Springer Verlag, 1981.

Whitham G.B., *Linear and Nonlinear Waves,* John Wiley and Sons, New York, 1974.

ASUPRA UNEI CLASE DE REȚELE CELULARE NEURALE FRACTALE PE BAZA POTENȚIALULUI FRACTAL ŞI IMPLICAȚIILE ACESTEIA ÎN PROCESUL DE CREŞTERE BACTERIALĂ

(Rezumat)

Considerând că în rețeaua neuronală din creier transmiterea impulsului nervos se realizează pe curbe continue, dar nediferențiabile (curbe fractale), introducem în varianta hidrodinamică a relativității de scală (cu dimensiune fractală arbitrară constantă) o clasă specială de rețele celulare neurale la scară fractală. Prin urmare, soluția de „energie” fractală infinită a modelului implică prin potențialul fractal moduri de oscilație cnoidală ale densității stărilor fractale. Aceste moduri pot fi asociate unei rețele fractale unidimensionale Toda şi, prin mapare, unei rețele celulare neurale fractale. Soluția cu „energie” fractală finită generată printr-un mecanism de rupere spontană de simetrie induce elemente de logică fractală cum ar fi, bitul fractal, porțile fractale etc. Se dă un exemplu asupra rolului jucat de codurile de comunicare ce sunt fundamentate prin logica fractală în procesul de creştere celulară.

BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 62 (66), Numărul 1, 2016
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

LETTER TO THE EDITORS of the Bulletin of the Polytechnic Institute of Iaşi, Series Mathematics. Theoretical Mechanics. Physics

Years ago (1965-1975), I was a beneficiary of the journal that you coordinate now, it was where I made my debut as an author and then I published some sections from my doctoral thesis, or continuations of it. For this reason, I have always followed the content of this journal. I was very proud of it (as an author) because many important names in Mathematics, in Iaşi and elsewhere, had works published by your journal, such as Acad. Dimitrie Mangeron, Prof. Dr. Alexandru Climescu, Prof. Dr. Ion Creangă, Prof. Dr. Adolf Haimovici, Prof. Dr. Olga Costinescu etc. From a certain point forward, my age and various other issues prevented me from having a direct contact with this satisfaction, but I have always been convinced that the Bulletin of the Polytechnic Institute of Iaşi keeps its high standards. However, to my disappointment and sadness, some time ago I noticed that something had changed, and not for the good. It may have been a mere accident, I would really like to think that, but its reason should be discussed in order to avoid things like this in the future. I will explain. In Fasc. 1 from 2015 there is an article, "Another proof for Fermat's Last Theorem", by Mugur Răuţ, Public Service Society, Iaşi, România. This title could have had a huge impact even on a middle school student who knows Pythagoras' theorem, and of course, the fact that this theorem was the starting point for the famous conjecture from the theory of numbers known as Fermat's Great (or Last) Theorem. However, it appears that such article did not create any emotion on the reviewer of the article or even the editor. The proof of Fermat's Last Theorem in 1995 by Andrew Wiles, the English mathematician who received one of the most important awards for his achievement (the Fermat Award – 1995, the Abel Award – 2016, etc.), was largely covered by the media, it appeared on TV, and even novels were written on the topic (e.g. "Fermat's Last Theorem", Simon Singh, 1997). The conjecture was formulated by the French mathematician Pierre de Fermat (1601-1665) in 1637. During the 358 years until it was demonstrated, surely tens of thousands of pages were written by famous mathematicians such as Euler (1753), Dirichlet, Legendre, Sophie Germain, Gerd Faultings (1983) ..., extraordinary awards were offered (the French Academy Award, the Paul Wolfskehl Award, etc.), strong computers were used to check the conjecture until n < 4,000,000 (!), intermediate conjectures were issued (Tanyama-Shimura, Mazur) and complicated mathematical theories were developed (elliptic curves, modularity, p-adic Galois representation, Hecke algebras, etc.), which were used in the final paper of Andrew Wiles. This work, [1], is 108 pages long, of which only the bibliography covers more than four pages and includes 83 titles, only for the recent period (approx. 1965-1994). And not only that. As a completion, in the same edition of Annals of Math, another 19-page article, [2], is published! Then, only a summary of the three Cambridge conferences (June 1993) where Wiles announced the proof of the Fermat Conjecture is 24 pages long [3], and an article of G. Faultings (winner of the Fields award) [4] analyses the essential part of the proof. Apparently, Mr. Mugur Răuț, the author, knew about all the above, if we are to look into the bibliography of his article [5]. Nevertheless, he dared refute all that I have mentioned here (very briefly) in the 7 pages of his article! Nevertheless, maybe this could happen to a young author (and by young I refer to his publishing experience), although I have never heard of such a case! I still do not see any excuse for the publication in the journal of this article, submitted on 11 February 2015 and accepted (amazingly!) on 9 March 2015. Let us bear in mind that even in the least rigorous of journals, and whoever the author may be, and with a title far from any suspicion, an article waits at least three months until it is published, and sometimes one year or even more, if the editor cannot find the suitable reviewer or if the originality or the accuracy is uncertain.

Finally one last remark: although I am not a physicist or astrophysicist, starting only from the comments above, the abstracts and the incredibly short reception-acceptation periods, I consider that articles [6] and [7] belonging to the same author are also strange.

Hoping that my letter to the Editors will help avoiding such unpleasant incidents in the future, I remain your faithful and humble reader of the Bulletin of the Polytechnic Institute in Iaşi, Series Mathematics. Theoretical Mechanics. Physics.

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Bibliography

1. Wiles A., *Modular Elliptic Curves and Fermat's Last Theorem*, Annals of Mathematics, 142, 443-551 (1995).
2. Taylor R., Wiles A., *Ring-Theoretic Properties of Certain Hecke Algebras*, Annals of Mathematics, 142, 553-572 (1995).
3. Rubin K., Silverberg A., *A Report on Wiles'Cambridge Lectures*, Bulletin of the American Math. Soc., Vol. 31, Number 1, July 1994, 15-38.
4. Faltings G., *The Proof of Fermat's Last Theorem* by R. Taylor and A. Wiles, Notices, July 1995, 743-746.
5. Răuț M., *Another Proof for Fermat's Last Theorem*, Bul. Inst. Polit. Iaşi, s. Mat. Mec. Teor. Fizică, *1*, 73-80 (2015).
6. Răuț M., *Bernoulli's Law for an Adiabatic Ideal Gas Flow*, Bul. Inst. Polit. Iaşi, s. Mat. Mec. Teor. Fizică, *2*, 107-112 (2015).
7. Răuț M., *Dark Matter and Dynamics of Galaxies:a Non-Newtonian Approach*, s. Mat. Mec. Teor. Fizică, *4*, 21-31 (2015).